Variance Change in Coincident Indicators of Japan

Takeshi OTSU

"ECONOMIC PAPERS" No. 251 (December, 2025)
The Economic Institute of Seijo University

Variance Change in Coincident Indicators of Japan

Takeshi OTSU

Abstract

This paper statistically investigates whether regime shifts exist in the variation of the coincident indicators of Japan. We use a Markov-switching model (Hamilton, 1989) and the test statistics proposed by Carrasco et al. (2014a). The main findings are as follows. First, the statistical tests reject the null hypothesis of constant variance across the coincident indicators of Japan for the periods 1975-2020 and 2008-2020. Second, the results of the hypothesis tests are mixed for the subsample periods 1975-1991 and 1991-2008. The null hypothesis of constant variance is rejected for Non-Scheduled Worked Hours, Retail Sales, Operating Profits and Exports Volume in both periods, but it is accepted for other series, such as the industrial production and the producer's shipments, in one of the periods. Finally, the expected duration of the low-variance state is longer than that of the high-variance state for all indicators.

Key words : Sup-type statistics, Exponential statistics, Nuisance parameter, Markov-switching model

JEL classification : C24

1 Introduction

We have observed a sharp reduction in output volatility in the U.S. after the mid-1980s. This phenomenon is known as the Great Moderation, as documented in the literature (Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000; Stock and Watson, 2003). It suggests that the variance of output, as well as its trend component, should be allowed to change in econometric models. Carrasco, Hu, and Ploberger (2014a) first conducted a formal statistical test on the hypothesis of the variance reduction in output, proposing tractable testing tools. They used a Markov-switching model (see Hamilton, 2016) that allows switching in both mean and variance, and found that the variance parameter constancy was rejected for the U.S. GNP data. The analysis presented in Otsu (2025) showed similar results with real GDP data of Japan.

To conduct statistical testing for the existence of switching effects with Markov-switching models, we cannot apply the standard asymptotic distribution theory because some parameters are unidentified under the null hypothesis. That is, the existence of unidentified nuisance parameters makes it impossible to consistently estimate model parameters because the likelihood function has multiple local optima and some elements of the score vector (the first-order derivatives) are identically zeros under the null hypothesis. The seminal works on the problem of unidentified nuisance parameters are provided by Davies (1977, 1987).

Carrasco et al. (2014a) addressed the issue of unidentified nuisance parameters by developing an optimal test in the sense of the Neyman-Pearson lemma. This approach only requires estimation of constant parameters of a model under the null hypothesis, which is similar to a Lagrange-Multiplier (LM) test. Closely-related tests have been proposed by Hansen (1992), Hansen (1996), Garcia (1998), and Cho and White (2007). Carrasco et al. (2014a) showed that their tests were comparable to or better than those competing tests in terms of test size and power. More recently, Qu and Zhuo (2021) used a higher-order approximation to refine the asymptotic null distribution of a *quasi*-likelihood ratio (QLR) test statistic, which can improve the power of the tests.

This paper statistically investigates whether regime shifts exist in the variation of Japan's coincident indicators. We use a regime-switching model (Hamilton, 1989) and the test statistics proposed by Carrasco et al. (2014a), namely, the supTS and the expTS. The main findings are as follows. First, the statistical tests reject the null hypothesis of constant variance across the coincident indicators of Japan for the periods 1975-2020 and 2008-2020. Second, the results of the hypothesis tests are mixed for the subsample periods 1975-1991 and 1991-2008. The null hypothesis of constant variance is rejected for Non-Scheduled Worked Hours, Retail Sales, Operating Profits and Exports Volume in both periods, but it is accepted for other series, such as the industrial production and the producer's shipments, in one of the periods. Finally, the expected duration of the low-variance state is longer than that of the high-variance state for all indicators.

The rest of the paper is organized as follows. In Section 2, we briefly discuss a regime-switching model and the test statistics proposed by Carrasco et al. (2014a). Section 3 summarizes the empirical results. The final section is devoted to discussion.

2 Testing Change in Variance

2.1 Model and Hypothesis

Let y_t an economic time-series variable at time $t\,(t = 1, 2, \cdots, T)$. We consider a simple regime-switching model as follows:

$$y_t = \mu + \mu_d S_t + u_t, \tag{1}$$

$$\phi(L)u_t = (\sigma + \sigma_d S_t)e_t, \tag{2}$$

$$e_t \sim N(0, 1), \tag{3}$$

$$\phi(L) = 1 - \sum_{j=1}^{K} \phi_j L^j, \quad L^k y_t \equiv y_{t-k}, \tag{4}$$

where μ, μ_d, σ, σ_d and ϕ_j are all unknown constant parameters. K is set to 4 in Hamilton (1989) and Carrasco et al. (2014a). In our later analysis, K takes values from 1 to 4. S_t denotes the economic state at time t, taking either 1 or 0. The transition between states is assumed to be governed by a first-order Markov process that is independent of e_t. Then, the transition probabilities are:

$$P[S_t = 1|S_{t-1} = 1] = p, \qquad 0 < p < 1, \tag{5}$$

$$P[S_t = 0|S_{t-1} = 0] = q, \qquad 0 < q < 1. \tag{6}$$

Here, it is possible to express the Markov chain in the form:

$$S_t = \rho S_{t-1} + \nu_t, \tag{7}$$

where ν_t is a martingale difference (Hamilton, 1994, p.679). We presume ν_t follows an i.i.d. uniform distribution, U[-1,1], and $-1 < \rho < 1$. Thus, S_t has bounded support $[-1/(1-|\rho|), 1/(1-|\rho|)]$ and has mean zero. Note that eq.(5) and eq.(6) imply that $\rho = p + q - 1$.

When μ_d and σ_d are zero, the model reduces to a single-state model. There are three types of hypotheses. One hypothesis is that each state has a different mean under the alternative hypothesis, assuming that the variance is same in both states:

$$H_0 : \mu_d = 0, \quad H_1 : \mu_d \neq 0, \tag{8}$$

where H_0 denotes the null hypothesis and H_1 the alternative hypothesis. An alternative null hypothesis is that only the variance parameter, σ, switches between the two states:

$$H_0 : \sigma_d = 0, \quad H_1 : \sigma_d > 0. \tag{9}$$

This is the hypothesis we use later in the paper. Finally, the null hypothesis can take the form that both the mean and variance are constant across the two states:

$$H_0 : \mu_d = 0 \text{ and } \sigma_d = 0, \quad H_1 : \mu_d \neq 0 \text{ or } \sigma_d > 0. \tag{10}$$

One might use the conventional t-statistic for testing, but it lacks the standard null distribution. Neither do other conventional statistics, such as the likelihood ratio and the chi-square statistics. This is because p and q are unidentified under the null hypothesis. That is, we cannot find unique estimates for these parameters to maximize the likelihood function. Further, the scores with respect to μ_d, p and q are identically zeros under the null hypothesis. Thus, the standard distribution theory is inapplicable.

2.2 Test Statistics

Following Carrasco et al. (2014a), we assume that the likelihood function for the model, consisting of eq.(1) to eq.(7), factorizes into two conditional likelihood functions: one for observed variable, y_t, given identified parameters under H_0, $\theta = (\mu, \sigma, \phi_1, \ldots, \phi_K)$, and another for a vector of latent variables, η_t, given the nuisance parameters $\beta = (\mu_d, \sigma_d, \rho)$. Both variables are stationary. Specifically, we have:

$$\mathcal{L}_T(\theta, \beta) = \prod_{t=1}^{T} f_t(y_t|y_{t-1}, y_{t-2}, \cdots, y_1; \theta) q_t(\eta_t|\eta_{t-1}, \eta_{t-2}, \cdots, \eta_1; \beta). \qquad (11)$$

Then, y_t and η_t are mutually independent under the null hypothesis, but there still exists a distribution of η_t. To derive the specific form of the test statistic for the model in subsection 2.1, η_t is assumed to be expressed as chS_t. Here, c is a scalar, indicating the amplitude of the state change, and h is a vector specifying the direction of the alternative change. For identification, the Frobenius norm of h is normalized to one, that is, $||h|| = 1$.

Let $\hat{\theta}$ the constrained maximum likelihood estimator of θ under H_0, and $l_t(\cdot)$ the log of $f_t(\cdot)$. Then, the test statistic, proposed by Carrasco et al. (2014a), is given by:

$$TS_T(\beta,\hat{\theta}) = \mathit{\Gamma}_T - \frac{1}{2T}\hat{\varepsilon}(\beta)'\hat{\varepsilon}(\beta), \tag{12}$$

where

$$\mathit{\Gamma}_T = \frac{1}{\sqrt{T}}\sum_t \mu_{2,t}(\beta,\hat{\theta}), \tag{13}$$

$$\begin{aligned}\mu_{2,t}(\beta,\hat{\theta}) = &\frac{1}{2}c^2h'\left\{\left(l_t^{(2)} + l_t^{(1)}l_t^{(1)\prime}\right)\right.\\ &\left.+\,2\sum_{s<t}\rho^{(t-s)}\left(l_t^{(1)}l_s^{(1)\prime}\right)\right\}h,\end{aligned} \tag{14}$$

$$l_t^{(1)}(\theta) = \frac{\partial l_t(\theta)}{\partial\theta}, \quad l_t^{(2)}(\theta) = \frac{\partial^2 l_t(\theta)}{\partial\theta\partial\theta'}, \tag{15}$$

and $\hat{\varepsilon}$ is a vector of the residuals from the OLS regression of $\mu_{2,t}(\beta,\hat{\theta})$ on $l_t^{(1)}(\hat{\theta})$. Carrasco et al. (2014a, b) showed that $\mathit{\Gamma}_T$ converges to a Gaussian process with zero mean, and $\hat{\varepsilon}(\beta)'\hat{\varepsilon}(\beta)/T$ converges to the asymptotic variance of $\mathit{\Gamma}_T$ that might depend on the nuisance parameters. Furthermore, they showed that this test statistic gives the most powerful test by the Neyman-Pearson lemma.

To derive the specific form of the test statistic for the model in subsection 2.1, the component of η_t is assumed to be expressed as chS_t. Here, c is a scalar, indicating the amplitude when the state changes, and h is a vector specifying the direction of the alternative change. For identification, the Frobenius norm of h is normalized to one, that is, $||h|| = 1$.

One of the tests used in Carrasco et al. (2014a) is the sup-type test as in Davies (1987). First, we maximize eq.(12) with respect to c^2 and use the solution to substitute out c^2. We divide $\mu_{2,t}$ by c^2 to make it free of c^2, and denote $\mu^*_{2,t}(\beta,\theta) = \mu_{2,t}(\beta,\theta)/c^2$ and $\mathit{\Gamma}^*_T = \sum \mu^*_{2,t}(\beta,\hat{\theta})/\sqrt{T}$. Then, the test statistic is:

$$\sup TS = \sup_{h,\rho} \frac{1}{2} \left(\max \left(0, \frac{\Gamma_T^*}{\sqrt{\hat{\varepsilon}^{*\prime} \hat{\varepsilon}^*}} \right) \right)^2, \tag{16}$$

where $\hat{\varepsilon}^*$ is a vector of the residuals from the OLS regression of $\mu_{2,t}^*(\beta, \hat{\theta})$ on $l_t^{(1)}(\hat{\theta})$. The parameter ρ takes a preset range $\underline{\rho} < \rho < \overline{\rho}$, denoted by $[\underline{\rho}, \overline{\rho}]$. The range of ρ is set to $[-0.7, 0.7]$ in Carrasco et al. (2014a). In the later analysis, we set $[\underline{\rho}, \overline{\rho}] = [-0.9, 0.9]$ to compute the test statistics because we find estimates of p and q take values of 0.8 to 0.9 in experiments, which are inconsistent with the range, $[-0.7, 0.7]$.

When the model in the previous subsection has two switching parameters, the mean and the variance, h takes a form of $(h_1, \mathbf{0}, h_2)'$, where the number of the size of the vector $\mathbf{0}$ is equal to the number of the autoregressive parameters. If only the variance switches, h takes the value of one at the position of the variance term and zeros at all other positions: $(0, \mathbf{0}, 1)'$.

Another test used in Carrasco et al. (2014a) is the (average) exponential-type test as in Andrews and Ploberger (1994, 1996). To evaluate an exponential statistic, we average out all nuisance parameters with some prior distributions. Following Carrasco et al. (2014a), an exponential prior, $\tau e^{-\tau c^2}$, is used for c^2 in eq.(14) to be bounded from above and to give an analytical solution for the integral of $\exp(TS_T(\beta, \hat{\theta}))$ with respect to c^2. As for h and ρ, uniform priors are used. Then, the (average) exponential statistic is given by:

$$\exp TS = \int_{\{||h|| \leq 1,\ \underline{\rho} < \rho < \overline{\rho}\}} \Psi(h, \rho) d\rho dh, \tag{17}$$

where

$$\Psi(h, \rho) = \begin{cases} \left(\frac{\tau\sqrt{2\pi}}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^*}} \right) exp \left[\frac{(\Gamma_T^* - \tau)^2}{2\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^*} \right] \Phi \left(\frac{\Gamma_T^* - \tau}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^*}} \right), & \text{if } \hat{\varepsilon}^{*\prime}\hat{\varepsilon}^* \neq 0, \\ 1, & \text{otherwise.} \end{cases} \tag{18}$$

In practice, we set $\tau = \sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}$ and obtain:

$$\Psi\left(h,\rho\right) = \begin{cases} \sqrt{2\pi} exp\left[\frac{1}{2}\left(\frac{\Gamma_T^*}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}}-1\right)^2\right]\Phi\left(\frac{\Gamma_T^*}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}}-1\right), & \text{if } \hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*} \neq 0, \\ 1, & \text{otherwise.} \end{cases} \tag{19}$$

Carrasco et al. (2014a) showed that each of these statistics weakly converges to a linear combination of Gaussian processes. We employ both statistics in eq.(16) and eq.(17). Following Carrasco et al. (2014a), we approximate the critical values and p-values of these statistics by parametric bootstrapping, or the Monte Carlo simulation. Recently, Amengual, Fiorentini, and Sentana (2024) argued that the parametric bootstrap provides reliable finite-sample size control and good power for finite Gaussian mixtures.

3 Empirical Results

We use the coincident indicators of the Composite Indexes, complied by Economic and Social Research Institute (ESRI) of the Cabinet Office, Government of Japan. ESRI routinely examines and revises the composition of these indicators. We use the 12threvision data set to enhance comparison with Otsu (2024), which used the same data set to test the mean-parameter switching based on the standardized likelihood ratio test statistic proposed by Hansen (1992).

The coincident indicators consist of ten series covering 1975 to 2020 (see Table 1). We examine not only the whole sample period but also the subsample periods, splitting at the asset bubble burst in 1991 and the 2007-2008 financial crisis. The variable y_t in eq.(1) is given by the difference in logarithm times 100 of each indicators, except the Retail Sales value (C6) and the Wholesale Sales value (C7) in Table 1 that are originally measured as a rate of change. Thus, the unit is percent for all variables.

To compute the statistics, we set $[\underline{\rho}, \overline{\rho}] = [-0.9, 0.9]$. To obtain the critical values and p-values, we conduct the parametric bootstrapping, or the Monte Carlo simulation. We use the random generator of the SFMT Mersenne-Twister 19937 (GAUSS software) to generate samples of i.i.d. normal observations. The number of replications is 3000 for the hypotheses in eq.(9) as in Carrasco et al. (2014a).

The test results are shown in Table 2 through Table 11. Note that we omit the detailed results for the period of March 2008 to December 2020 to save space. We observe that the expTS tends to give a larger value than the supTS. All of the statistics reject the null hypothesis $H_0 : \sigma_d = 0$ for each indicator at the conventional significance levels, 5% or 1%, during the full sample period of January 1975 December 2020.

For the subsample periods, the test outcomes depend on the indicator used for the periods of January 1975 to February 1991 and March 1991 to February 2008, while the null hypothesis is rejected for all series during the periods of March 1991 to December 2020 and March 2008 to December 2020. We summarize these results in Table 12 for the supTS statistic.

In the period of January 1975 to February 1991, the null hypothesis $H_0 : \sigma_d = 0$ is rejected for Whole Sales (C7), Operating Profits (C8), Effective Job Offer Rate(C9), and Exports Volume (C10), while it is not rejected at the conventional significance levels for Industrial Production (C1) and the producer's shipments of Producer Goods (C2), Durable Consumer Goods (C3) and Investment Goods (C5). The null hypothesis is marginally rejected for Non-Scheduled Worked Hours (C4) and Retail Sales (C6).

In contrast, the null hypothesis is rejected for Industrial Production (C1) in the period of March 1991 to February 2008, whereas it is not rejected for Whole Sales (C7) and Effective Job Offer Rate(C9). In addition, it is rejected for Retail Sales (C6) when the statistical model includes the autoregressive terms up to the fourth order. Finally, the null hypothesis is rejected at the conventional significance level for the producer's shipment of Durable Consumer Goods (C3).

We find a shift in the variance of Operating Profits (C8) and Exports Volume (C10) across both the entire sample and subsample periods, but there is no evidence of a change in the variance of the producer's shipment of Investment Goods (C5) until March 2008.

Since we find shifts in the variance during the periods of January 1975 to December 2020 and March 2008 to December 2020, we also estimate a Markov-switching model that allows shifts both in the mean and the variance. We set K in eq.(4) to one in order to reduce the computational burden. Table 14 and Table 15 show the estimation results for January 1975 to December 2020. As expected, the variance shifts between the high-variance and low-variance states across all series. The estimates are statistically significant at conventional significance levels.

The expected duration of the low-variance state is longer than that of the high-variance state. The high-variance state lasts more than 12 years for Industrial Production (C1) and Producer Goods (C2), while it lasts approximately 10 years for Durable Consumer Goods (C3). The low-variance state of these series continues about 10 months. The longest duration of the low-variance state is estimated for Non-Scheduled Worked Hours (C4) : approximately 18 years. The duration of the high-variance state of Non-Scheduled Worked Hours (C4) is about 6 months. The low-variance state continues for nearly 3 years for Investment Goods (C5) and just over 6 years for Effective Job Offer Rate(C9). The duration of the high-variance state for these series is about 6 months and one year and three months, respectively.

The duration of the low variance remains between 1.5 and 2 years for Retail Sales (C6), Whole Sales (C7), and Exports Volume (C10). The duration of the high-variance state ranges from 3 months to 5 months. Operating Profits (C8) gives rise to the shortest durations of both the low-variance and high-variance states – approximately 2 months and 1.5 months, respectively. This implies that Operating Profits (C8) would exhibit cycles of less than four months, which is inconsistent with the conventional understanding of the business cycle. Therefore, this indicator may not be appropriate for constructing the coincident composite index.

Table 16 and Table 17 show the estimation results for March 2008 to December 2020. We find that the variance shifts between high and low states across all series, and the estimates are statistically significant at conventional levels. However, the durations of both the low-variance and high-variance states shorten for six series: Industrial Production (C1), Producer Goods (C2), Durable Consumer Goods (C3), Non-Scheduled Worked Hours (C4), Retail Sales (C6), and Whole Sales (C7). As for Investment Goods (C5), the duration of the high-variance state shortens from about three years to one year, whereas the duration of the low-variance state slightly increases from 6 months to 7 months. The duration of the low-variance state of Effective Job Offer Rate(C9) shortens from approximately 6 years to just over 4 years, while that of the high-variance state increases from one year and three months to approximately 4 years. Thus, the gap between the durations of the two states narrows. Similarly, the durations of the low-variance and high-variance states become increasingly similar for Operating Profits (C8), approaching two months. The longest duration is about four years and the shortest is two months across the states during the period from March 2008 to December 2020.

4 Discussion

This paper statistically investigates whether regime shifts exist in variation of the coincident indicators of Japan. We use a regime-switching model (Hamilton, 1989) and the test statistics proposed by Carrasco et al. (2014a), namely, the supTS and the expTS. The main findings are as follows. First, the statistical tests reject the null hypothesis of constant variance across the coincident indicators of Japan for the periods 1975-2020 and 2008-2020. Comparing with the results of the hypothesis testing on the constant mean in Table 13, the Markov-switching effects are more pronounced in the variance shift.

Second, the hypothesis test results are mixed for the subsample periods 1975-1991 and 1991-2008. The null hypothesis of constant variance is rejected for Non-Scheduled Worked Hours (C4), Retail Sales (C6), Operating Profits (C8) and Exports Volume (C10) in both periods, but it is accepted for other series, such as the industrial production and the producer’s shipments, in one of the periods.

Finally, the expected duration of the low-variance state is longer than that of the high-variance state for all indicators. This sharply contrasts with the estimates of Otsu (2025) for real GDP, which suggest an expected duration of approximately 15 years for the low-variance state and over 25 years for the high-variance state for the period of 1955 to 2001. However, they also suggest that the expected duration of the low-variance state is longer for the period of 1994 to 2024 – approximately 10 years versus 9 months.

A couple of caveats are worth noting. First, the estimation results show that the estimates of the constant term of the Markov-switching model are not statistically significant. Thus, there is no evidence of a shift in the mean. Otsu (2024) uses the same dataset to estimate a similar two-state Markov-switching model under the assumption of constant variance, and finds that the mean estimates are statistically significant in each state. Therefore, further investigation into the model specification is warranted. Second, a statistical test of the joint null hypothesis that both the mean and variance remain constant is necessary for completeness. In this paper, the computational burden prevented us from conducting such a test.

Finally, the test statistics used in the paper explicitly account for the serial correlation of regime shifts, but they overlook the effects of asymmetry and tail behavior in the mixture distribution, which could enhance the power of the tests. Qu and Zhuo (2021) addressed this issue and proposed an alternative test statistic, which is applicable to our setting. These are subjects for future research.

Acknowledgements **Part of this work was financially supported by** *Tokubetsu Kenkyu Josei* **(Seijo University) under the project titled**「経済変動の離散的変化に関する検証」**(2024–2025**).

References

Amengual, D., G. Fiorentini, E. Sentana, 2024, "The information matrix test for Gaussian mixtures," presented at 2025 Allied Social Science Association Annual Meeting, San Francisco, CA, January 5th.

Andrews, D.W. K. and W. Ploberger, 1994, "Optimal tests when a nuisance parameter is present only under the alternative," Econometrica 62, 1383-1414.

Andrews, D.W. K. and W. Ploberger, 1996, "Testing for serial correlation against an ARMA(1, 1) process," Journal of the American Statistical Association 91, 1331-1342.

Carrasco, M., L. Hu, and W. Ploberger, 2014a, "Optimal test for Markov switching parameters," Econometrica 82, 765-784.

Carrasco, M., L. Hu, and W. Ploberger, 2014b, "Supplement to 'Optimal Test for Markov Switching Parameters'," Econometrica Supplemental Material 82.

Cho, J. S. and White, H., 2007, "Testing for regime switching," Econometrica 75, 1671-1720.

Davies, R.B., 1977, "Hypothesis testing when a nuisance parameter is present only under the alternative," Biometrika 64, 247-254.

Davies, R.B., 1987, "Hypothesis testing when a nuisance parameter is present only under the alternative," Biometrika 74, 33-34.

Garcia, R., 1998, "Asymptotic null distribution of the likelihood ratio test in Markov switching models," International Economic Review 39, 763-788.

Hamilton, J.D., 1989, "A new approach to the economic analysis of nonstationary time series and the business cycle," Econometrica 57, 357-384.

Hamilton, J.D., 1994, Time Series Analysis, Princeton University Press, New Jersey.

Hamilton, J.D., 2016, "Macroeconomic Regimes and Regime Shifts," Ch.3., In Handbook of Macroeconomics, Volume 2A, edited by John B. Taylor and Harald Uhlig. Amsterdam: Elsevier, 163-201.

Hansen, B. E., 1992, “The likelihood ratio test under non-standard conditions: Testing the Markov switching model of GNP,” Journal of Applied Econometrics 7, S61-S82. Erratum, 1996, 11, 195-198.

Hansen, B. E., 1996, “Inference when a nuisance parameter is not identified under the null hypothesis,” Econometrica 64, 413-430.

Kim, C.-J., and C. R. Nelson, 1999, “Has the U.S. economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle,” The Review of Economics and Statistics 81, 608-616.

McConnell, M. M., and G. Perez-Quiros, 2000, “Output fluctuations in the United States: What has changed since the early 1980’s?” American Economic Review 90, 1464-1476.

Otsu, T., 2024, “State Change in Coincident Indicators,” Seijo University Economic Papers 244, 59-94, The Economic Institute of Seijo University (Japan).

Otsu, T., 2025, “Shift in Variance of Real GDP,” Seijo University Economic Papers 247・248, 137-153, The Economic Institute of Seijo University (Japan).

Qu, Z. and F. Zhuo, 2021, “Likelihood ratio-based tests for Markov regime switching,” The Review of Economic Studies 88, 937-968.

Stock, J.H., and M.W. Watson, 2003, “Has the business cycle changed and why?,” in: M. Gertler and K. Rogoff, eds., NBER Macroeconomics Annual 2002, Vol. 17, MIT Press, Cambridge, 159-218.

Table 1 Coincident Indicators of Japan (the 12th Revision*)

Label	Series Name	Sample Period
C1	Index of Industrial Production (Mining and Manufacturing),	Jan.1975 - Dec. 2020.
C2	Index of Producer's Shipments (Producer Goods for Mining and Manufacturing)	Jan.1975 - Dec. 2020.
C3	Index of Producer's Shipment of Durable Consumer Goods	Jan.1975 - Dec. 2020.
C4	Index of Non-Scheduled Worked Hours (Industries Covered)	Jan.1975 - Dec. 2020.
C5	Index of Producer's Shipment (Investment Goods Excluding Transport Equipment)	Jan.1975 - Dec. 2020.
C6	Retail Sales Value (Change From Previous Year,%)	Jan.1975 - Dec. 2020.
C7	Wholesale Sales Value (Change From Previous Year,%)	Jan.1975 - Dec. 2020.
C8	Operating Profits (All Industries, 100 Mil. Yen)**	Jan.1975 - Sept. 2020.
C9	Effective Job Offer Rate (excl. New School Graduates, Times: # of job offers / # of active job seekers.)	Jan.1975 - Dec. 2020.
C10	Exports Volume Index	Jan.1975 - Dec. 2020.

* Monthly data. Feb. 25th 2021, published by Economic and Social Research Institute, Japan.
** Only quarterly series are available. A linear interpolation is used to obtain monthly series.
† The base year of each index is 2015.

Table 2 Test Results of $H_0 : \sigma_d = 0$; C1(Industrial Production: Mining and Manufacturing)

Number	Type of	Jan.1975 - Dec. 2020		Before Feb. 1991		After Mar. 1991	
of lags (K)	statistic	Stat.	p-value	Stat.	p-value	Stat.	p-value
1	supTS	25.463	0.000	0.581	0.275	22.332	0.000
	expTS	18729764.611	0.000	1.003	0.190	1399595.108	0.000
2	supTS	27.333	0.000	1.214	0.122	23.786	0.000
	expTS	85286130.775	0.000	1.477	0.073	4572272.095	0.000
3	supTS	27.224	0.000	1.711	0.069	22.672	0.000
	expTS	79499322.430	0.000	1.551	0.070	1932724.038	0.000
4	supTS	27.524	0.000	1.767	0.065	22.897	0.000
	expTS	104469574.508	0.000	1.664	0.060	2358898.084	0.000

Note: Number of replications to compute p-values is 3000.

Table 3 Test Results of $H_0 : \sigma_d = 0$; C2(Producer Goods for Mining and Manufacturing)

Number	Type of	Jan.1975 - Dec. 2020		Before Feb. 1991		After Mar. 1991	
of lags (K)	statistic	Stat.	p-value	Stat.	p-value	Stat.	p-value
1	supTS	29.994	0.000	1.239	0.119	25.531	0.000
	expTS	167344×10^4	0.000	1.568	0.062	3697×10^4	0.000
2	supTS	33.942	0.000	1.287	0.104	29.356	0.000
	expTS	5602569×10^4	0.000	1.564	0.060	89503×10^4	0.000
3	supTS	37.818	0.000	2.552	0.026	31.572	0.000
	expTS	129033239×10^4	0.000	2.625	0.019	481193×10^4	0.000
4	supTS	39.027	0.000	2.691	0.020	31.531	0.000
	expTS	418305228×10^4	0.000	2.629	0.014	448831×10^4	0.000

Note: Number of replications to compute p-values is 3000.

Table 4 Test Results of $H_0 : \sigma_d = 0$; C3(Durable Consumer Goods)

Number	Type of	Jan.1975 - Dec. 2020		Before Feb. 1991		After Mar. 1991	
of lags (K)	statistic	Stat.	p-value	Stat.	p-value	Stat.	p-value
1	supTS	36.623	0.000	0.867	0.186	32.106	0.000
	expTS	4940×10^8	0.000	1.199	0.132	106×10^8	0.000
2	supTS	59.737	0.000	1.034	0.160	43.055	0.000
	expTS	117418276×10^{12}	0.000	1.286	0.112	720299×10^8	0.000
3	supTS	51.181	0.000	1.200	0.134	39.123	0.000
	expTS	497283679×10^8	0.000	1.372	0.099	22571×10^8	0.000
4	supTS	50.357	0.000	1.036	0.141	38.859	0.000
	expTS	363878768×10^8	0.000	1.227	0.124	24403×10^8	0.000

Note: Number of replications to compute p-values is 3000.

Table 5 Test Results of $H_0 : \sigma_d = 0$; C4(Non-Scheduled Worked Hours)

Number	Type of	Jan.1975 - Dec. 2020		Before Feb. 1991		After Mar. 1991	
of lags (K)	statistic	Stat.	p-value	Stat.	p-value	Stat.	p-value
1	supTS	22.784	0.000	2.306	0.027	24.105	0.000
	expTS	3607857.416	0.000	1.949	0.030	10999910.300	0.000
2	supTS	19.606	0.000	3.156	0.007	19.084	0.000
	expTS	326365.533	0.000	3.270	0.005	195108.299	0.000
3	supTS	20.037	0.000	3.468	0.008	19.816	0.000
	expTS	456036.963	0.000	3.976	0.005	349481.597	0.000
4	supTS	19.671	0.000	3.325	0.011	19.740	0.000
	expTS	565058.535	0.000	3.645	0.006	458739.170	0.000

Note: Number of replications to compute p-values is 3000.

Table 6 Test Results of $H_0 : \sigma_d = 0$; C5(Investment Goods excl. Transport Equipment)

Number of lags (K)	Type of statistic	Jan.1975 - Dec. 2020 Stat.	p-value	Before Feb. 1991 Stat.	p-value	After Mar. 1991 Stat.	p-value
1	supTS	14.354	0.000	1.046	0.135	11.320	0.000
	expTS	5733.982	0.000	1.250	0.105	525.930	0.000
2	supTS	14.727	0.000	1.522	0.073	11.215	0.000
	expTS	8469.438	0.000	1.601	0.057	493.612	0.000
3	supTS	15.620	0.000	1.430	0.094	11.366	0.000
	expTS	14029.909	0.000	1.482	0.076	513.054	0.000
4	supTS	18.679	0.000	1.359	0.099	13.970	0.000
	expTS	127911.268	0.000	1.436	0.080	2858.988	0.000

Note: Number of replications to compute p-values is 3000.

Table 7 Test Results of $H_0 : \sigma_d = 0$; C6(Retail Sales Value)

Number of lags (K)	Type of statistic	Jan.1975 - Dec. 2020 Stat.	p-value	Before Feb. 1991 Stat.	p-value	After Mar. 1991 Stat.	p-value
1	supTS	28.057	0.000	5.254	0.001	39.438	0.000
	expTS	355×10^6	0.000	11.161	0.000	15141702×10^6	0.000
2	supTS	30.391	0.000	2.604	0.023	36.972	0.000
	expTS	7017×10^6	0.000	3.070	0.010	1109662×10^6	0.000
3	supTS	31.014	0.000	2.244	0.034	37.103	0.000
	expTS	7269×10^6	0.000	2.592	0.019	1197112×10^6	0.000
4	supTS	33.271	0.000	2.026	0.046	37.866	0.000
	expTS	54456×10^6	0.000	2.307	0.022	2502722×10^6	0.000

Note: Number of replications to compute p-values is 3000.

Table 8 Test Results of $H_0 : \sigma_d = 0$; C7(Retail Sales Value)

Number of lags (K)	Type of statistic	Jan.1975 - Dec. 2020 Stat.	p-value	Before Feb. 1991 Stat.	p-value	After Mar. 1991 Stat.	p-value
1	supTS	15.070	0.000	5.464	0.000	11.926	0.000
	expTS	5228.845	0.000	7.745	0.000	634.898	0.000
2	supTS	18.952	0.000	7.520	0.000	18.080	0.000
	expTS	199788.617	0.000	28.769	0.000	76474.547	0.000
3	supTS	20.889	0.000	8.383	0.000	16.625	0.000
	expTS	888210.941	0.000	44.376	0.000	20141.597	0.000
4	supTS	12.594	0.000	5.477	0.001	7.931	0.000
	expTS	2252.538	0.000	14.042	0.000	46.631	0.000

Note: Number of replications to compute p-values is 3000.

Table 9 Test Results of $H_0 : \sigma_d = 0$; C8(Operating Profits)

Number of lags (K)	Type of statistic	Jan.1975 - Dec. 2020 Stat.	p-value	Before Feb. 1991 Stat.	p-value	After Mar. 1991 Stat.	p-value
1	supTS	30.090	0.000	19.521	0.000	28.401	0.000
	expTS	32×10^8	0.000	569883.916	0.000	11×10^8	0.000
2	supTS	31.551	0.000	18.944	0.000	29.111	0.000
	expTS	103×10^8	0.000	377696.955	0.000	19×10^8	0.000
3	supTS	33.969	0.000	23.517	0.000	30.648	0.000
	expTS	616×10^8	0.000	10704351.332	0.000	61×10^8	0.000
4	supTS	41.499	0.000	16.560	0.000	40.018	0.000
	expTS	733697×10^8	0.000	13104.133	0.000	359648×10^8	0.000

Note: Number of replications to compute p-values is 3000.

Table 10 Test Results of $H_0 : \sigma_d = 0$; C9(Effective Job Offer Rate)

Number of lags (K)	Type of statistic	Jan.1975 - Dec. 2020 Stat.	p-value	Before Feb. 1991 Stat.	p-value	After Mar. 1991 Stat.	p-value
1	supTS	18.224	0.000	18.585	0.000	10.329	0.000
	expTS	34185.232	0.000	17977.345	0.000	276.595	0.000
2	supTS	16.715	0.000	15.774	0.000	11.258	0.000
	expTS	16289.063	0.000	3566.792	0.000	315.850	0.000
3	supTS	13.959	0.000	13.244	0.000	11.691	0.000
	expTS	1699.711	0.000	415.909	0.000	463.874	0.000
4	supTS	12.133	0.000	13.115	0.000	40.018	0.000
	expTS	3172.889	0.000	331.334	0.000	622.011	0.000

Note: Number of replications to compute p-values is 3000.

Table 11 Test Results of $H_0 : \sigma_d = 0$; C10 (Exports Volume Index)

Number of lags (K)	Type of statistic	Jan.1975 - Dec. 2020 Stat.	p-value	Before Feb. 1991 Stat.	p-value	After Mar. 1991 Stat.	p-value
1	supTS	13.382	0.000	11.639	0.000	13.886	0.000
	expTS	767.919	0.000	740.051	0.000	1062.170	0.000
2	supTS	17.303	0.000	12.973	0.000	16.970	0.000
	expTS	30368.722	0.000	1872.434	0.000	52185.832	0.000
3	supTS	18.284	0.000	12.370	0.000	18.451	0.000
	expTS	206375.018	0.000	1595.955	0.000	196033.180	0.000
4	supTS	18.843	0.000	15.751	0.000	18.033	0.000
	expTS	189636.433	0.000	17755.857	0.000	138633.065	0.000

Note: Number of replications to compute p-values is 3000.

Table 12 Range of p - values: supTS

Period	p - value $< 1\%$	$1\% \leq p$ - value $< 5\%$	$5\% \leq p$ - value
Jan.1975- Feb.1991	C4(Non-sched. ;K=2,3) C6(Retail Sales;K=1) C7(Whole Sales) C8(Operating Profits) C9(Job Offer) C10(Exports)	C4(Non-sched. ;K=1,4) C6(Retail Sales;K=2,3,4) C2(Prod.Goods;K=3,4)	C1(Production) C3(Durable Goods) C2(Prod.Goods;K=1,2) C5(Investment Goods)
Mar.1991- Feb.2008	C1(Production) C3(Durables;K=1,2) C6(Retail Sales) C8(Operating Profits) C10(Exports)	C2(Producer Goods) C3(Durables;K=3,4) C4(Non-sched. Worked)	C5(Investment Goods) C7(Whole Sales) C9(Job Offer)
Mar.2008- Dec.2020*	All series for K=1 to 4.		

Note: * Sample period ends in September. 2020 for C8.

Table 13 p - Value of the Standardized LR statistic; Test Results of $H_0 : \mu_d = 0$

Period	p - value $< 1\%$	$1\% \leq p$ - value $< 5\%$	$5\% \leq p$ - value
Jan.1975- Dec.2020	C1(Production) C6(Retail Sales) C8(Operating Profits) C9(Job Offer)	C2(Producer Goods) C7(Whole Sales) C10(Exports)	C3(Durable Goods) C4(Non-sched. Worked) C5(Investment Goods)
Jan.1975- Feb.1991	C6(Retail Sales)	C2(Producer Goods) C5(Investment Goods) C9(Job Offer)	C1(Production) C3(Durable Goods) C4(Non-sched. Worked) C7(Whole Sales) C8(Operating Profits) C10(Exports)
Mar.1991- Dec.2020*	C1(Production) C2(Producer Goods) C6(Retail Sales) C8(Operating Profits) C10(Exports)	C9(Job Offer)	C3(Durable Goods) C4(Non-sched. Worked) C5(Investment Goods) C7(Whole Sales)

Note: * Sample period ends in September 2020 for C8.
Excerpt from Otsu (2024), Table 4.

Table 14 Markov-Switching Model: Jan.1975 - Dec.2020 ($K = 1$)

Parameter*	C1	C2	C3	C4	C5
$\mu + \mu_d$	0.16928	0.20752	0.27899	0.08467	0.24183
(std.error)	(0.04611)	(0.05377)	(0.09386)	(0.05216)	(0.08124)
μ	-0.20944	-0.43674	-0.34292	-2.58951	-0.90729
(std.error)	(0.78313)	(1.57859)	(0.48157)	(1.95902)	(0.44015)
ϕ_1	-0.30236	-0.15402	-0.32934	0.05425	-0.29336
(std.error)	(0.04928)	(0.04665)	(0.05571)	(0.04396)	(0.04592)
$\sigma + \sigma_d$	1.27145	1.33427	2.59392	1.09676	1.81167
(std.error)	(0.05031)	(0.04798)	(0.13732)	(0.04262)	(0.10146)
σ	6.81112	6.98649	14.61150	6.43621	4.13017
(std.error)	(1.08307)	(0.76243)	(3.04176)	(1.58922)	(0.52849)
p	0.99338	0.99358	0.99017	0.99535	0.97073
(std.error)	(0.00389)	(0.00338)	(0.00578)	(0.00353)	(0.01380)
q	0.89196	0.90735	0.90821	0.82678	0.84524
(std.error)	(0.05873)	(0.04283)	(0.00578)	(0.07812)	(0.07276)
LL value**	-983.74928	-1016.76064	-1411.17663	-871.68750	-1211.95015
# of obs.	550	550	550	550	550
Expected Durations of High and Low Variances (month)					
Low Var.	151	156	102	215	34
High Var.	9	11	11	6	6

Note: The dependent variable is the rate of change (%).
* "std.error": Heteroskedastic consistent estimates.
** "LL value": Log-Likelihood value.

Table 15 Markov-Switching Model: Jan.1975 - Dec.2020 ($K = 1$)

Parameter*	C6	C7	C8	C9	C10
$\mu + \mu_d$	2.32493	0.97078	-3.00767	0.38081	0.29421
(std.error)	(0.84669)	(2.14695)	(1.06687)	(0.20482)	(0.07456)
μ	1.61685	0.65723	-3.00538	-2.00949	0.11591
(std.error)	(0.60772)	(2.24225)	(1.06758)	(1.80535)	(0.24344)
ϕ_1	0.86901	0.91581	0.992245	0.67577	-0.23242
(std.error)	(0.02067)	(0.02152)	(0.00220)	(0.04524)	(0.04804)
$\sigma + \sigma_d$	5.78121	4.94051	0.02955	1.29049	1.84858
(std.error)	(0.92300)	(0.67895)	(0.00268)	(0.05038)	(0.09806)
σ	1.56718	2.24023	7.13956	3.04386	6.64489
(std.error)	(0.09300)	(0.14935)	(2.44338)	(0.40718)	(0.925933)
p	0.72302	0.80307	0.52981	0.98670	0.95751
(std.error)	(0.14386)	(0.10128)	(0.02870)	(0.01031)	(0.01511)
q	0.94522	0.94646	0.34492	0.93359	0.76590
(std.error)	(0.01622)	(0.02712)	(0.03307)	(0.07873)	(0.07096)
LL value**	-1211.37041	-1356.89300	-455.00940	-1018.33992	-1282.52785
# of obs.	550	550	547†	550	550
Expected Durations of High and Low Variances (month)					
Low Var.	18	19	2	75	24
High Var.	4	5	2	15	4

Note: The dependent variable is the rate of change (%).
* "std.error": Heteroskedastic consistent estimates.
** "LL value": Log-Likelihood value.
† Sample period ends in September 2020 for C8.

Table 16 Markov-Switching Model: Mar.2008 - Dec.2020 ($K = 1$)

Parameter*	C1	C2	C3	C4	C5
$\mu + \mu_d$	0.10964	0.29359	-0.08568	0.12659	0.31807
(std.error)	(0.16643)	(0.20304)	(0.32766)	(0.12085)	(0.17321)
μ	-1.00857	-4.37704	-1.68090	-2.61989	-1.18642
(std.error)	(1.95084)	(2.29015)	(3.51076)	(1.88062)	(0.55985)
ϕ_1	-0.14183	0.19040	0.05273	0.07935	-0.22996
(std.error)	(0.17406)	(0.07214)	(0.08096)	(0.06511)	(0.08665)
$\sigma + \sigma_d$	1.42522	1.71256	3.21535	1.10538	1.92987
(std.error)	(0.15013)	(0.13394)	(0.26607)	(0.13924)	(0.21464)
σ	6.75989	7.22044	16.86839	6.30863	4.71297
(std.error)	(1.09805)	(0.93209)	(2.67860)	(1.83780)	(0.42590)
p	0.97165	0.97398	0.96385	0.98123	0.92483
(std.error)	(0.01822)	(0.01333)	(0.01626)	(0.01598)	(0.04237)
q	0.85188	0.79152	0.81138	0.81861	0.86137
(std.error)	(0.11307)	(0.08704)	(0.10242)	(0.07790)	(0.08370)
LL value**	-323.23619	-337.51281	-450.18199	-269.23329	-378.30563
# of obs.	152	152	152	152	152
Expected Durations of High and Low Variances (month)					
Low Var.	35	38	28	53	13
High Var.	7	5	5	6	7

Note: The dependent variable is the rate of change (%).
* "std.error": Heteroskedastic consistent estimates.
** "LL value": Log-Likelihood value.

Table 17 Markov-Switching Model: Mar.2008 - Dec.2020 ($K = 1$)

Parameter*	C6	C7	C8	C9	C10
$\mu + \mu_d$	0.65239	-1.17511	3.33543	0.81795	0.12718
(std.error)	(0.33067)	(1.05486)	(3.75263)	(0.25143)	(0.23098)
μ	0.25827	-0.33458	3.35603	-1.63557	-0.78497
(std.error)	(1.38464)	(0.85542)	(3.75358)	(0.85511)	(1.73843)
ϕ_1	0.58468	0.93183	1.01331	0.70787	-0.25657
(std.error)	(0.05423)	(0.03892)	(0.01606)	(0.07629)	(0.15180)
$\sigma + \sigma_d$	1.33169	1.84996	0.07784	0.77323	2.34181
(std.error)	(0.11020)	(0.36040)	(0.03374)	(0.07669)	(0.22635)
σ	6.79078	5.05443	12.66701	2.10894	8.95546
(std.error)	(0.94342)	(0.62827)	(4.87796)	(0.28098)	(1.40135)
p	0.93022	0.81204	0.56062	0.98101	0.97112
(std.error)	(0.03012)	(0.10063)	(0.05750)	(0.01696)	(0.01910)
q	0.65850	0.71045	0.47332	0.98020	0.87681
(std.error)	(0.15903)	(0.07937)	(0.10135)	(0.03392)	(0.06699)
LL value**	-327.95011	-393.84541	-271.84730	-239.53589	-396.08910
# of obs.	152	152	149†	152	152
Low Var.	14	5	2	53	35
High Var.	3	3	2	51	8

Note: The dependent variable is the rate of change (%).
* "std.error": Heteroskedastic consistent estimates.
** "LL value": Log-Likelihood value.
† Sample period ends in September 2020 for C8.