Shift in Variance of Real GDP

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Abstract

This paper statistically investigates whether regime shifts exist in variation of real GDP growth data of Japan. We use a regime-switching model (Hamilton, 1989) and the test statistics proposed by Carrasco et al. (2014a). The main findings are as follows. First, the tests reject the null of variance constancy for the real GDP growth. Output volatility switches between a low and a high state for the entire period of 1955 afterwards. Second, the mean constancy is not rejected when the variance is assumed constant, especially when the model under the null hypothesis has the third- and the fourth-order autoregressive terms. Finally, the expected duration of staying in the high-volatility state decreases from 25 years before 2001 to 2 or 3 years in the recent period.

Key words : Sup-type Statistics, Exponential Statistics, Nuisance Parameter, Markov switching Model

JEL classification : C24

1 Introduction

In the empirical studies of the business cycle, a great attention has been paid to the change in the trends, or the rate of change, of economic variables. As pointed out by Stock and Watson (2003), however, the striking change in the business cycle in the U.S. is the sharp reduction in output volatility after the mid-1980s. This is the so-called great moderation, as documented in the literature (Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000). It suggests that the variance of the output, as well as the trend component, should be allowed to change in econometric models. This hypothesis had not been formally tested until Carrasco, Hu, and Ploberger (2014a) proposed tractable testing tools. They used a Markov-switching model to apply their testing method for regime switching in mean and variance to the U.S. GNP data, and found that the parameter consistency was rejected.

Markov-switching models are widely used in the literature to capture switching effects in the model parameters (see Hamilton, 2016). When we conduct statistical testing for the existence of switching effects with these models, we cannot apply the standard asymptotic distributional theory because some parameters are unidentified under the null hypothesis. That is, the existence of unidentified nuisance parameters makes it impossible to consistently estimate model parameters because the likelihood function has multiple local optima and some of the elements of the score vector (the first-order derivatives) are identically zeros under the null hypothesis. This is the problem of unidentified nuisance parameters, which was analyzed by Davies (1977, 1987).

Carrasco et al. (2014a) dealt with the problem of unidentified nuisance parameters by developing an optimal test in the sense of the Neyman-Pearson lemma. It only requires estimation of constant parameters of a model under the null hypothesis, which is similar to a Lagrange-Multiplier (LM) test. Closely-related tests have been proposed by Hansen (1992), Hansen (1996), Garcia (1998), and Cho and White (2007). Carrasco et al. (2014a) showed that their tests were comparable or better than those competing tests in terms of sizes and powers of the tests. More recently, Qu and Zhuo (2021) used a higher-order approximation to refine the asymptotic null distribution of a *quasi*-likelihood ratio (QLR) test statistic, which can improve the power of the tests.

This paper statistically investigates whether regime shifts exist in variation of real GDP growth data of Japan. We use a regime-switching model (Hamilton, 1989) and the test statistics proposed by Carrasco et al. (2014a), that is, the supTS and the expTS. To the best of my knowledge, there is little research examining regime switching in mean and variance using Japanese data.

The main findings are as follows. First, the tests reject the null of variance constancy for the real GDP growth. Output volatility switches between a low and a high state for the entire period of 1955 afterwards. Second, the mean constancy is not rejected when the variance is assumed constant, especially when the model under the null hypothesis has the third- and the fourth-order autoregressive terms. Finally, the expected duration of staying in the high-volatility state decreases from 25 years before 2001 to 2 or 3 years recently, while that of the low-volatility state remains around 10 years through the sample periods.

The rest of the paper is organized as follows. In section 2, we briefly discuss the testing hypotheses to be studied and review the test statistics proposed by Carrasco et al. (2014a). Section 3 summarizes the empirical results. The final section is allocated to discussion.

2 Testing Hypothesis and Test Statistic

2.1 Testing Hypothesis

Let y_t an economic time-series variable at time $t\,(t=1,2,\cdots,T)$. We consider a simple regime-switching model as follows:

$$y_t = \mu + \mu_d S_t + u_t, \tag{1}$$

$$\phi(L)u_t = (\sigma + \sigma_d S_t)e_t, \tag{2}$$

$$e_t \sim N(0,1), \tag{3}$$

$$\phi(L) = 1 - \sum_{j=1}^{K} \phi_j L^j, \quad L^k y_t \equiv y_{t-k} \tag{4}$$

where μ, μ_d, σ, σ_d and ϕ_j are all unknown constant parameters. K is set to 4 in Hamilton (1989) as well as in Carrasco et al. (2014a). In our later analysis, K takes from 1 to 4. S_t denotes the economic state at time t, taking either 1 or 0. The transition between states is assumed to be governed by a first-order Markov process that is independent of e_t. Then, the transition probabilities are:

$$P[S_t = 1 | S_{t-1} = 1] = p, \qquad 0 < p < 1, \tag{5}$$

$$P[S_t = 0 | S_{t-1} = 0] = q, \qquad 0 < q < 1. \tag{6}$$

Here, it is possible to express the Markov chain in the form:

$$S_t = \rho S_{t-1} + \nu_t, \tag{7}$$

where ν_t is a martingale difference (Hamilton, 1994, p.679). We presume ν_t follows an i.i.d. uniform distribution, U[-1,1], and $-1 < \rho < 1$. Thus, S_t has bounded support $[-1/(1-|\rho|), 1/(1-|\rho|)]$ and has mean zero. Note that eq.(5) and eq.(6) imply that $\rho = p + q - 1$.

When μ_d and σ_d are zero, this model reduces to one-state model. We consider three hypotheses sets as follows to see if the two-state model is appropriate. One of them is that each state has a different mean under the alternative hypothesis, assuming that the variance is same in both states:

$$H_0 : \mu_d = 0, \quad H_1 : \mu_d \neq 0, \tag{8}$$

where H_0 denotes the null hypothesis and H_1 the alternative one. Another is that only variance parameter, σ, switches between the two states:

$$H_0 : \sigma_d = 0, \quad H_1 : \sigma_d > 0. \tag{9}$$

In the final case, the alternative hypothesis is that each state is different in either mean or variance:

$$H_0 : \mu_d = 0 \text{ and } \sigma_d = 0, \quad H_1 : \mu_d \neq 0 \text{ or } \sigma_d > 0. \tag{10}$$

One might use the conventional t-statistic for testing, but it lacks the standard null distribution. Neither do other conventional statistics, such as the likelihood ratio or the chi-square statistics. This is because p and q are unidentified under the null hypothesis. That is, we cannot find unique estimates for these parameters to maximize the likelihood function. Further, the scores with respect to μ_d, p and q are identically zeros under the null hypothesis. Then, the standard distributional theory is inapplicable.

2.2 Test Statistics

Following Carrasco et al. (2014a), we assume that the likelihood function for the model, consisting of eq.(1) to eq.(7), factorizes as two conditional likelihood functions: one for observed variable, y_t, given identified parameters under H_0, $\theta = (\mu, \sigma, \phi_1, \ldots, \phi_K)$, and another for a vector of latent variables, $\eta_\mathbf{t}$, given the nuisance parameters $\beta = (\mu_d, \sigma_d, \rho)$. Both variables are stationary. Specifically, we have:

$$\mathcal{L}_T\,(\theta\,, \beta) = \prod_{t=1}^{T} f_t\,(y_t | y_{t-1}, y_{t-2}, \cdots, y_1; \theta) q_t(\eta_\mathbf{t} | \eta_\mathbf{t-1}, \eta_\mathbf{t-2}, \cdots, \eta_\mathbf{1}; \beta). \tag{11}$$

Then, y_t and $\eta_\mathbf{t}$ are mutually independent under the null hypothesis, but there still exists a distribution of $\eta_\mathbf{t}$. Let $\hat{\theta}$ the constrained maximum likelihood estimator of θ under H_0, and $l_t(\cdot)$ the log of $f_t(\cdot)$. Then, the test statistic, propsed by Carrasco et al. (2014a), is written as:

$$TS_T(\beta, \hat{\theta}) = \mathit{\Gamma}_T - \frac{1}{2T}\hat{\varepsilon}(\beta)'\hat{\varepsilon}(\beta), \tag{12}$$

where

$$\mathit{\Gamma}_T = \frac{1}{\sqrt{T}} \sum_t \mu_{2,t}(\beta, \hat{\theta}), \tag{13}$$

$$\begin{aligned}\mu_{2,t}(\beta, \theta) = \frac{1}{2} \Big\{ & trace \left(\left(l_t^{(2)} + l_t^{(1)} l_t^{(1)\prime} \right) E \left(\eta_{\mathrm{t}}(\beta) \eta_{\mathrm{t}}{}'(\beta) \right) \right) \\ & + 2 \sum_{s<t} trace \left(l_t^{(1)} l_s^{(1)\prime} E \left(\eta_{\mathrm{t}}(\beta) \eta_{\mathrm{t}}{}'(\beta) \right) \right) \Big\},\end{aligned} \tag{14}$$

$$l_t^{(1)} = \frac{\partial l_t(\theta)}{\partial \theta}, \quad l_t^{(2)} = \frac{\partial^2 l_t(\theta)}{\partial \theta \partial \theta'}, \tag{15}$$

and $\hat{\varepsilon}$ is a vector of the residuals from the OLS regression of $\mu_{2,t}(\beta, \hat{\theta})$ on $l_t^{(1)}(\hat{\theta})$. Carrasco et al. (2014a) showed that $\mathit{\Gamma}_T$ converges to a Gaussian process with zero mean, and $\hat{\varepsilon}(\beta)'\hat{\varepsilon}(\beta)/T$ converges to the asymptotic variance of $\mathit{\Gamma}_T$ that might depend on the nuisance parameters. Furthermore, they showed that this test statistic gives the most powerful test by the Neyman-Pearson lemma.

To derive the specific form of the test statistic for the model in subsection 2.1, the component of η_{t} is assumed to be expressed as chS_t. Here, c is a scalar, indicating the amplitude when the state changes, and h is a vector specifying the direction of the alternative change. For identification, the Frobenius norm of h is normalized to one, that is, $||h_t|| = 1$. In this case, eq.(14) can be written as:

$$\begin{aligned}\mu_{2,t}(\beta, \hat{\theta}) = \frac{1}{2} c^2 h' \Big\{ & \left(l_t^{(2)} + l_t^{(1)} l_t^{(1)\prime} \right) \\ & + 2 \sum_{s<t} \rho^{(t-s)} \left(l_t^{(1)} l_s^{(1)\prime} \right) \Big\} h.\end{aligned} \tag{16}$$

One of the tests used in Carrasco et al. (2014a) is the sup-type test as in Davies (1987). First, we maximize eq.(12) with respect to c^2 and use the solution to substitute out c^2. We divide $\mu_{2,t}$ by c^2 to make it free of c^2, and denote $\mu_{2,t}^*(\beta, \theta) = \mu_{2,t}(\beta, \theta)/c^2$ and $\mathit{\Gamma}_T^* = \sum \mu_{2,t}^*(\beta, \theta)/\sqrt{T}$. Then, the test statistic is:

$$\text{sup}TS = \sup_{h,\rho} \frac{1}{2}\left(\max\left(0, \frac{\Gamma_T^*}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^*}}\right)\right)^2, \tag{17}$$

where $\hat{\varepsilon}^*$ is a vector of the residuals from the OLS regression of $\mu_{2,t}^*(\beta,\hat{\theta})$ on $l_t^{(1)}(\hat{\theta})$. The parameter ρ takes a preset range $\underline{\rho} < \rho < \overline{\rho}$, denoted by $[\underline{\rho}, \overline{\rho}]$. The range of ρ is set to $[-0.7, 0.7]$ in Carrasco et al. (2014a). In the later analysis, we set $[\underline{\rho}, \overline{\rho}] = [-0.9, 0.9]$ to compute the test statistics because we find estimates of p and q take values of 0.8 to 0.9 in experiments, which are inconsistent with the range, $[-0.7, 0.7]$.

When only one parameter of the model switches, the vector h contains one at that position. For example, in the case of the model from the previous subsection, when only the constant term (the mean) switches, h takes the value of one at the position of the constant term and zeros at all other positions: $(1, \mathbf{0})'$. When the model has two switching parameters, the mean and the variance, h takes a form of $(h_1, \mathbf{0}, h_2)'$, where the number of the size of the vector $\mathbf{0}$ is equal to the number of the autoregressive parameters.

Another test used in Carrasco et al. (2014a) is the (average) exponential-type test as in Andrews and Ploberger (1994, 1996). To evaluate an exponential statistic, we average out all nuisance parameters with some prior distributions. Following Carrasco et al. (2014a), an exponential prior, $\tau e^{-\tau c^2}$, is used for c^2 in eq.(16) to be bounded from above and to give an analytical solution for the integral of $exp(TS_T(\beta,\hat{\theta}))$ with respect to c^2. As for h and ρ, uniform priors are used. Then, the (average) exponential statistic is given by:

$$\text{exp}TS = \int_{\{||h||\leq 1,\ \underline{\rho}<\rho<\overline{\rho}\}} \Psi(h,\rho) d\rho dh, \tag{18}$$

where

$$\Psi(h,\rho)=\begin{cases}\left(\frac{\tau\sqrt{2\pi}}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}}\right)exp\left[\frac{(\Gamma_T^{*}-\tau)^2}{2\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}\right]\Phi\left(\frac{\Gamma_T^{*}-\tau}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}}\right), & \text{if } \hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}\neq 0,\\ 1, & \text{otherwise.}\end{cases} \tag{19}$$

In practice, we set $\tau=\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}$ and obtain:

$$\begin{aligned}&\Psi\,(h,\rho)\\ &=\begin{cases}\sqrt{2\pi}exp\left[\frac{1}{2}\left(\frac{\Gamma_T^{*}}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}}-1\right)^2\right]\Phi\left(\frac{\Gamma_T^{*}}{\sqrt{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}}}-1\right), & \text{if } \hat{\varepsilon}^{*\prime}\hat{\varepsilon}^{*}\neq 0,\\ 1, & \text{otherwise.}\end{cases}\end{aligned} \tag{20}$$

Carrasco et al. (2014a) showed that each of these statistics weakly converges to a linear combination of Gaussian processes. We use both statistics in eq.(17) and eq.(18). Following Carrasco et al. (2014a), we approximate the critical values and p-values of these statistics by parametric bootstrapping, or the Monte Carlo simulation. Recently, Amengual, Fiorentini, and Sentana (2024) argue that the parametric bootstrap provides reliable finite sample sizes and good power for finite Gaussian mixtures.

3 Empirical Results

We use three series of real GDP data of Japan as shown in Table 1 because the System of National Account (SNA) is revised several times since 1947. Each series provides as many observations as possible for the same framework. y_t in eq.(1) is given by the difference of real GDP in logarithm times 100, so that the unit is percent.

To compute the statistics, we set $[\underline{\rho},\overline{\rho}]=[-0.9,0.9]$. In addition, for the hypotheses in eq.(10), we randomly generate 100 vectors of h, so that the Frobenius norm of h is normalized one: $||h_t||=1$. To obtain the critical values and p-values, we conduct the parametric bootstrapping, or the Monte Carlo simulation. We use the random generator of the SFMT Mersenne-Twister 19937 (GAUSS software) to generate samples of i.i.d. normal observations. The number of replications is 3000 for the hypotheses in eq.(8) and eq.(9) as in Carrasco et al. (2014a). But, it is 1000 for the hypotheses in eq.(10) due to the limited computational resources available to our research. In experiment, we conducted 1500 replications for some cases and found the test results, specifically p-values, did not change. Therefore, our conclusion is unlikely to change regardless of the number of replications.

Table 2 shows the results of testing $H_0 : \mu_d = 0$. When the model under the null has the second-order AR terms $(K = 2)$ at most, both tests reject the null hypothesis at conventional significance levels. When the third-order AR term is included in the null model, the linear AR model is not rejected. The same holds true for the model with AR terms up to the fourth order. Therefore, the mean growth rate can be modeled as a simple linear AR model for the period after 1955. This result does not depend on the difference of SNA definition.

In contrast, we find that the linear AR models are rejected when we test $H_0 : \sigma_d = 0$ as shown in Table 3. Similarly, when we test $H_0 : \mu_d = 0$ and $\sigma_d = 0$ against $H_1 : \mu_d \neq 0$ or $\sigma_d > 0$, the linear models with constant parameters are rejected, as indicated in Table 4. This result remains robust across lag lengths of the AR terms, definition of SNA, and sample periods.

We also estimate a regime-switching model that allows switching in both mean and variance. For the period of 1955Q2 to 2001Q1, the high-volatility state comes with a large positive growth rate on average, while the low-volatility state shows a small positive growth rate, as found in Table 5. The estimates of the transition probabilities imply that the expected duration of the high-volatility state is around 25 years, and that of the low-volatility state is 12.5 years.

When we use 1993SNA data of 1980Q1 to 2010Q4, we find that both states show small average growth rates, ranging from 0.43% to 0.66% in Table 6. This result is similar to that from the U.S. data reported in Carrasco et al. (2014b, p.11). The low-volatility state is expected to last approximately 10 years, while the high-volatility state is expected to last around 2 years. Turning to the results from 2008SNA data of 1994Q1 to 2024Q3 in Table 7, we find a negative average growth rate in the high-volatility state and a small positive growth rate in the low-volatility state. The expected duration is about 10 years for the low volatility state and 3 years for the high one, respectively.

To sum up, the switching effect is more evident in variance than in mean. The mean growth rate divides the states into a low positive and a high positive growth state before 2001, and into a low positive and a negative growth state in more recent data. Furthermore, the economy remained in the high-volatility state for an average of 25 years before 2001, but only for 2 to 3 years when more recent data is included in the analysis. The expected duration of the low volatility is around 10 to 12 years through the sample periods.

4 Discussion

This paper statistically investigates whether regime shifts exist in variation of real GDP growth data of Japan. We use a regime-switching model (Hamilton, 1989) and the test statistics proposed by Carrasco et al. (2014a), that is, the supTS and the expTS. The main findings are as follows. First, the tests reject the null of variance constancy for the real GDP growth. Output volatility switches between a low and a high state for the entire period of 1955 afterwards. Second, the mean constancy is not rejected when the variance is assumed constant, especially when the model under the null hypothesis has the third- and the fourth-order autoregressive terns. Then, a simple AR model is not rejected. Finally, the expected duration of staying in the high-volatility state decreases from 25 years before 2001 to 2 or 3 years in the recent period, while that of the low-volatility state remains approximately 10 years through the sample periods.

A couple of caveats are worth noting. First, the test statistics used in the paper explicitly account for the serial correlation of regime shifts, but they overlook the effects of asymmetry and tail behavior of the mixture distribution that might help the improvement of testing powers. Qu and Zhuo (2021) addressed this issue and proposed a likelihood-ratio-type test taking these effects into account. Therefore, we need to examine whether the test results change if we apply their testing method. Secondly, we need to examine a broad range of economic variables, such as leading indicators, coincident indicators, and the composite indicators to confirm the existence of shifts in variance. Finally, computational burden limits number of replications in simulation. Therefore, there may be room to enhance the accuracy of estimates for the test statistics and the corresponding p-values. We might need to make our algorithm more efficient. These are subjects for future research.

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Table 1 Real Gross Domestic Product of Japan by different versions of SNA *

Label	Series Name	Sample Period
RGDP68	Real GDP (expenditure, seasonally adjusted series) 1968SNA series ; Benchmark year=1990 (billion yen) (Apr.-Jun.1999 to Jan.-Mar.2000: Preliminary) (On and after Apr.-Jun. 2000: Referential) Released on Jun. 21st, 2001.	1955 Q2 - 2001 Q1.
RGDP93	Real GDP (expenditure, seasonally adjusted series) 1993SNA series ; Billions of chained (2000) yen. Released on Mar. 10th, 2011.	1980 Q1 - 2010 Q4.
RGDP08	Real GDP (expenditure, seasonally adjusted series) 2008SNA series ; Billions of chained (2015) yen. Released on Dec. 9th, 2024.	1994 Q1 - 2024 Q3.

* Quarterly data. Published by Economic and Social Research Institute, Japan.

Table 2 Test Results of $H_0 : \mu_d = 0$

Number	Type of	1955Q2 - 2001Q1		1980Q1 - 2010Q4		1994Q1 - 2024Q3	
of lags (K)	statistic	Stat.	p-value	Stat.	p-value	Stat.	p-value
1	supTS	19.059	0.000	2.692	0.034	0.712	0.388
	expTS	171353.505	0.000	2.168	0.031	0.935	0.200
2	supTS	6.262	0.002	2.356	0.082	4.242	0.012
	expTS	6.064	0.005	1.079	0.172	1.291	0.091
3	supTS	1.257	0.306	1.510	0.239	2.074	0.114
	expTS	0.745	0.649	0.956	0.292	1.151	0.133
4	supTS	0.530	0.602	1.213	0.255	0.005	0.976
	expTS	0.706	0.839	0.976	0.245	0.554	0.889

Note: Number of replications to compute p-values is 3000.

Table 3 Test Results of $H_0 : \sigma_d = 0$

Number of lags (K)	Type of statistic	1955Q2 - 2001Q1 Stat.	1955Q2 - 2001Q1 p-value	1980Q1 - 2010Q4 Stat.	1980Q1 - 2010Q4 p-value	1994Q1 - 2024Q3 Stat.	1994Q1 - 2024Q3 p-value
1	supTS	6.517	0.000	5.360	0.001	16.086	0.000
	expTS	12.850	0.000	11.559	0.000	18294.425	0.000
2	supTS	6.913	0.000	5.666	0.001	15.498	0.000
	expTS	50.406	0.000	13.651	0.000	14546.159	0.000
3	supTS	9.823	0.000	6.514	0.000	15.256	0.000
	expTS	102.516	0.000	20.511	0.000	11860.232	0.000
4	supTS	9.426	0.000	6.553	0.000	17.964	0.000
	expTS	112.018	0.000	22.047	0.000	94450.086	0.000

Note: Number of replications to compute p-values is 3000.

Table 4 Test Results of $H_0 : \mu_d = 0$ and $\sigma_d = 0$

Number of lags (K)	Type of statistic	1955Q2 - 2001Q1 Stat.	1955Q2 - 2001Q1 p-value	1980Q1 - 2010Q4 Stat.	1980Q1 - 2010Q4 p-value	1994Q1 - 2024Q3 Stat.	1994Q1 - 2024Q3 p-value
1	supTS	19.544	0.000	7.358	0.005	16.117	0.000
	expTS	12660.527	0.000	8.882	0.003	1251.960	0.000
2	supTS	7.460	0.008	7.413	0.009	15.867	0.000
	expTS	17.994	0.001	8.934	0.005	4578.666	0.000
3	supTS	10.208	0.000	8.355	0.005	15.550	0.000
	expTS	33.745	0.001	14.557	0.000	3499.119	0.000
4	supTS	9.989	0.000	8.573	0.002	18.139	0.000
	expTS	49.228	0.000	14.935	0.000	25303.615	0.000

Note: Number of replications to compute p-values is 1000.

Table 5 Markov Switching Model: 1955Q2 - 2001Q1 (1968SNA)

Parameter*	Estimate	Estimate	Estimate	Estimate
$\mu + \mu_d$	1.62626	1.51845	1.37872	1.24208
(std.error)	(0.21937)	(0.25904)	(0.30790)	(0.42962)
μ	0.86601	0.87254	0.88282	0.91619
(std.error)	(0.08432)	(0.12269)	(0.15232)	(0.20271)
ϕ_1	0.14480	0.12416	0.0480109	0.02140
(std.error)	(0.09195)	(0.07998)	(0.08412)	(0.08498)
ϕ_2	—	0.30976	0.27670	0.22950
(std.error)	(—)	(0.07534)	(0.08437)	(0.09717)
ϕ_3	—	—	0.24982	0.26082
(std.error)	(—)	(—)	(0.06658)	(0.06825)
ϕ_4	—	—	—	0.14533
(std.error)	(—)	(—)	(—)	(0.08407)
$\sigma + \sigma_d$	1.59395	1.48400	1.43043	1.41094
(std.error)	(0.15469)	(0.12039)	(0.10996)	(0.11969)
σ	0.68396	0.62548	0.61322	0.61481
(std.error)	(0.05761)	(0.06160)	(0.06944)	(0.06762)
p	0.99164	0.99204	0.99196	0.99188
(std.error)	(0.00880)	(0.00712)	(0.00705)	(0.00644)
q	0.98304	0.98153	0.98145	0.98172
(std.error)	(0.01406)	(0.01277)	(0.01305)	(0.01325)
LL value**	-279.27264	-269.87938	-262.42091	-258.48072
# of obs.	182	181	180	179

Note: The dependent variable is the rate of change of real GDP (%).
* "std.error": Heteroskedastic consistent estimates.
** "LL value": Log-Likelihood value.

Table 6 Markov Switching Model: 1980Q1 - 2010Q4 (1993SNA)

Parameter*	Estimate	Estimate	Estimate	Estimate
$\mu + \mu_d$	0.50320	0.49035	0.42918	0.43863
(std.error)	(0.11511)	(0.12380)	(0.13197)	(0.12411)
μ	0.64332	0.56955	0.63217	0.65904
(std.error)	(0.70013)	(0.76353)	(0.55112)	(0.57122)
ϕ_1	0.19674	0.20783	0.17076	0.19153
(std.error)	(0.11281)	(0.11506)	(0.09799)	(0.10443)
ϕ_2	—	0.07250	0.02003	0.03340
(std.error)	(—)	(0.11338)	(0.07869)	(0.08789)
ϕ_3	—	—	0.23246	0.23238
(std.error)	(—)	(—)	(0.07668)	(0.08420)
ϕ_4	—	—	—	- 0.08865
(std.error)	(—)	(—)	(—)	(0.08407)
$\sigma + \sigma_d$	0.73809	0.73348	0.68627	0.68885
(std.error)	(0.07540)	(0.07195)	(0.05680)	(0.06235)
σ	2.06927	2.08969	2.08360	2.07227
(std.error)	(0.46883)	(0.45558)	(0.38332)	(0.38608)
p	0.96890	0.97391	0.97585	0.97547
(std.error)	(0.02370)	(0.01776)	(0.01578)	(0.01583)
q	0.87060	0.87029	0.88518	0.88151
(std.error)	(0.06248)	(0.06233)	(0.06290)	(0.06321)
LL value**	-169.63577	-165.69255	-159.89416	-158.55406
# of obs.	122	121	120	119

Note: The dependent variable is the rate of change of real GDP (%).
* "std.error": Heteroskedastic consistent estimates.
** "LL value": Log-Likelihood value.

Table 7 Markov Switching Model: 1994Q1 - 2024Q3 (2008SNA)

Parameter*	Estimate	Estimate	Estimate	Estimate
$\mu + \mu_d$	0.30521	0.29519	0.29933	0.29241
(std.error)	(0.07113)	(0.07277)	(0.06286)	(0.05688)
μ	- 1.14001	- 1.19220	- 0.68466	- 0.33425
(std.error)	(1.62773)	(1.91056)	(3.29545)	(0.86717)
ϕ_1	- 0.02338	- 0.01627	- 0.06589	- 0.04791
(std.error)	(0.23217)	(0.36717)	(0.71917)	(0.10078)
ϕ_2	—	0.01906	- 0.00111	- 0.05313
(std.error)	(—)	(0.20132)	(0.76490)	(0.07696)
ϕ_3	—	—	- 0.10209	- 0.12209
(std.error)	(—)	(—)	(0.06970)	(0.04823)
ϕ_4	—	—	—	- 0.09697
(std.error)	(—)	(—)	(—)	(0.04854)
$\sigma + \sigma_d$	0.72837	0.72484	0.71778	0.71210
(std.error)	(0.05476)	(0.07679)	(0.13611)	(0.05658)
σ	3.58406	3.57137	3.54460	3.75327
(std.error)	(0.96443)	(1.33733)	(2.28609)	(1.03763)
p	0.97638	0.97594	0.97656	0.97552
(std.error)	(0.01912)	(0.02108)	(0.01994)	(0.02048)
q	0.70742	0.71149	0.71741	0.68057
(std.error)	(0.17977)	(0.20255)	(0.28510)	(0.20184)
LL value**	-158.94033	-157.65243	-155.29191	-152.79117
# of obs.	121	120	119	118

Note: The dependent variable is the rate of change of real GDP (%).
* "std.error": Heteroskedastic consistent estimates.
** "LL value": Log-Likelihood value.