Abridged Bry-Boschan Procedure

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Abstract

This paper examines what kind of filtering would be useful for the Bry-Boschan (BB) procedure to replicate the official reference dates, using Index of Industrial Production (IIP) of Japan. We use the official seasonal-adjustment series, filtering-based equivalents, and business-cycle components extracted by bandpassing. The main findings are as follows. First, the business cycle components, extracted by bandpass filters, are useful for the BB procedure to produce dates of the turns close to the reference dates. Particularly, the Butterworth-based bandpass filtering works well. Second, the seasonally adjusted series, whether adjusted by X12-ARIMA (official data) or by a bandpass filter, cannot identify turns in 1960s, or overidentify those in 2000s. Finally, we find a series of the moving averages and outlier adjustment in the BB procedure are not useful to replicate the official reference dates when we use the tangent-based Butterworth filter. Then, we obtain an abridged edition of the BB procedure.

1 Introduction

Determination of turning points is essential for empirical analyses of the business cycles. It governs basic characteristics of the cycles such as durations, amplitudes, and synchronization among regions and industries. It could also influence subsequent analyses of economic modelling and plicy implications, since it gives some guidance or even criteria to evaluate fitness of economic models and effects of economic policies. It is common that governments organize an official committee to determine the reference dates of the business cycles. It judges the reference dates based on not only all the relevant economic variables but also other unquantifiable, qualitative economy-related information. The reference dates are publicly available on the internet for the United Sates and Japan.

Judgment of dating by the committee involves arbitrariness because interpretation of economic condition is subjective to some extent and dating rules vary from time to time. Then, it is desirable to reduce such arbitrariness for objective and systematic business-cycle analyses. Further, clarity of the decision processes would be helpful for revision of the dating rules later on. Such an attempt is made by Bry and Boschan (1971). It codified dating rules and considerations that are predicated on Burns and Mitchell (1946), that is, the NBER (National Bureau of Economic Research) method at that time, and developed a program-based procedure of turning points determination, which is called the Bry-Boschan (BB) procedure hereafter. Nowadays, official agencies do not depend solely on the BB procedure. But it is still useful for academic research to codify a dating rule. For example, Watson (1994) used the BB procedure to study the business cycles in the U.S. Its modified version is used in Pagan (1997), Harding and Pagan (2001), Harding and Pagan (2002), and Harding and Pagan (2006). The algorithm is now publicly available and even embedded in some softwares of economic analyses.

The BB procedure is developed based on the dating rule heuristically found with the U.S. data in the first half of the 20th century. It involves various kinds of moving averages, so that it could identify peaks and troughs as close as possible to those selected by the staff at NBER. It also sets some constraints on durations of the business cycles or phases. Therefore, when we apply the BB procedure to recent economic data, other country's data, or data at regional or industrial levels, we might need some adjustments to the procedure. This requires researchers to tackle two issues: find appropriate duration constraints and choose a suitable moving-average method. The former is a matter of economic analysis, that is, the business-cycle analysis itself, and the latter a matter of statistical analysis. However, it is hard to identify what kinds of moving averages, or filtering methods in general, should be used. There would be several statistical criteria, and different criteria lead to different moving averages. Even a chosen criterion does not designate a specific moving average. In fact, the choice of the moving averages in the BB procedure is arbitrary. Therefore, it would be desirable to reduce this arbitrariness or ambiguity and to fix a moving-average method suitable for the business-cycle analysis. Then, the duration information of the business cycle would be suffice to adjust the procedure to a particular analysis.

One way to reduce such arbitrariness is to allow specified business-cycle durations to determine all the weights of moving averages. This can be done with bandpass filtering. The bandpass filters attempt to extract certain cyclical components. Thus, once we decide a range of the business cycle, we could extract particular components that should include useful information to determine turning dates. The remaining problem is what filters should be used. To choose a filter among those proposed in the literature, we could use a criterion in line with Bry and Boschan (1971): how well the official reference dates are replicated. This criterion is widely used in the literature (e.g. Hamilton, 1989; Canova, 1994, 1998, 1999).

With these considerations, this paper examines which filtering method would be useful for the BB procedure to replicate the official reference dates of Japan, using Index of Industrial Production (IIP) of Japan. We use the official seasonal-adjustment series, filtering-based equivalents, and business-cycle components extracted by bandpass filters. We seek a filtering method involving no parameter estimation, so that all researchers need is to determine the periodicities of the cycles appropriate for their research. To my knowledge, there is not much research in the literature to examine an alternative choice of the moving averages or the duration rules for the BB procedure, whereas it is listed as 'SUGGESTIONS FOR FURTHER DEVELOPMENT' in Bry and Boschan (1971, p. 56). We attempt to fill this gap.

The main findings as follows. First, the business cycle components, extracted by bandpass filters, are useful for the BB procedure to produce dates of the turns close to the reference dates. Particularly, the Butterworth-based bandpass works well. Second, the seasonally adjusted series, whether adjusted by XI2-ARIMA (official data) or by a bandpass filter, cannot identify turns in 1960s, or overidentify those in 2000s. Finally, we find that the moving averages and outlier adjustment in the BB procedure are ineffective when we use the tangent-based Butterworth filter. Thus, we could abridge the BB procedure.

The remaining parts of the paper are organized as follows. In the next section, we briefly review the BB procedure. Section 3 summarizes the filtering methods used in the paper. Empirical results are presented in Section 4. Final discussion and remaining issues are given in Section 5.

2 Bry-Boshcan Procedure

The BB procedure is summarized in Table 1. Watson (1994) found some discrepancies between the original description by Bry and Boschan (1971) and the Fortran program they coded. The description here is modified to be consistent with the Fortran codes. The procedure presumes to use seasonally adjusted series. In Step I, outliers, if any, are replaced by the values of the Spencer curve. Here, outliers are defined as values whose ratios to (or differences in absolute value from, depending on data) the 15-point Spencer curve are larger than 3.5 standard deviations, the threshold value chosen arbitrarily. This Spencer curve is computed as the 15-month symmetric moving average with particular weights (see Kendall and Stuart, 1966, p. 458).

Step II starts with the 12-month moving average (MA12, hereafter) of the outlier-free series. The MA12 is chosen on the ground that the Spencer curve contains too many minor fluctuations. Any date with the highest value among the 6 preceding and the 6 following months is tentatively regarded as the date of a peak. Similarly, any date with the lowest among the 6 preceding and the 6 following months is considered the date of a tentative trough. These peaks and troughs are checked for alternation. For contiguous peaks or troughs, the highest value is chosen for a peak, and the lowest for a trough. If the values are same, we set an earlier date for a peak, and a later date for a trough, respectively.

In Step III, the Spencer curve of the outlier-free series is used to ensure peaks and troughs within ±6 months, because its turns are heuristically closer to those of the original series than those of MA12. If there are ties within ±6 data points on the Spencer curve, an earlier date is chosen for a peak, and a later date for a trough. After alternation check as in Step II, the durations of a peak to peak or a trough to trough (a full cycle) are enforced to be at least 15-month period. If the durations are too short, the lower of two peaks or the higher of two troughs are eliminated. If the values are same, we set an earlier date for a peak, and a later date for a trough, respectively. Alternation check is conducted if any modification.

In Step IV, a further refinement is conducted with a short-term moving average, which is called MCD (Months for Cyclical Dominance) curve. The MCD is obtained as follows. First, we compute the Spencer curve of the original series, taking it as the trend-cycle component. The difference between the original series and the trend-cycle component gives the irregular component. Next, we take the ratio of the average change in the irregular component to that in the trend-cycle component. The change is computed either by the rate of change or by the difference of each component over various time spans. The MCD is the minimum number of months that gives the ratio less than 1. That is, the MCD is the shortest period of months that it takes for the change in the trend-cycle component to dominate that in the irregular component. The BB procedure confines the MCD between 3 to 6 months. Then, a short-term moving average is computed over the span of MCD, and used to ensure peaks and troughs within ±6 months as in Step III. Alternation is checked as in Step II if modified.

In the final step ('V'), a series of tests are conducted to determine final turns. First, the original series is used to ensure peaks and troughs within ±4 months or ±4 MCD, whichever is longer (denoted by 'V.1'). The second test ('V.2') is alternation check as in Step II. Third ('V.3'), any turns closer than 6 months from the ends are removed. In the fourth test

('V.4'), if the first or the last peak (or trough) takes a smaller (or greater) value than any value between it and the end of the original series, it is removed. In the program used by Watson (1994), the first and the last turns are only compared with the initial and the last data points, respectively, not with all the values between them. Although this could make a nontrivial difference, it does not change the results of the paper. Here, we follow Watson (1994).

The fifth test ('V.5') is to check if the duration of a full cycle is at least 15-month length, as in Step III. The final test ('V.6') is to check whether a phase (peak to trough or trough to peak) duration is at least 5 months. If it is less than 5 months, the two turning points are eliminated. If the violation is found at the last turning point, only the last point is removed. In later experiments, we implement the procedure with various smoothed series in Step II instead of the 12-month moving average. Also, in some cases, we skip Step III, Step IV, and Step V.1 and V.2 to see their effects.

3 Extracting Business Cycle Components

If it is possible to extract business cycle components, we could use them in the BB procedure. The cyclical components, if properly extracted, would incorporate all the turns to be identified as peaks or troughs. Therefore, the BB procedure, specifically the moving average computation, would be simplified, or even some processes are omitted. In the literature, there are various methods to extract and measure cyclical information. Canova (2007) gives a concise description of methods frequently used in macroeconomic analyses. In this section, we review properties of three methods used later: Christiano-Fitzgerald filter, Hamming filter, and Butterworth filters. To begin with, we consider the following orthogonal decomposition of the observed series x_t :

$$x_t = y_t + \tilde{x}_t \tag{1}$$

where y_t is a signal whose frequencies belong to the interval $\{[-b, -a] \cup [a, b]\} \in [-\pi, \pi]$, while $\tilde{x}_t$ has the complementary frequencies. Suppose that we wish to extract the signal y_t. The Wiener-Kolmogorov theory of signal extraction, as expounded by Whittle (1983, Chapter 3 and 6), indicates y_t can be written
as:

$$y_t = B(L)x_t \tag{2}$$

$$B(L) = \sum_{j=-\infty}^{\infty} B_j L^j, \quad L^k x_t \equiv x_{t-k} \tag{3}$$

In polar form, we have

$$B(e^{-iw}) = \begin{cases} 1, \text{ for } \omega \in [-b, -a] \cup [a, b] \\ 0, \text{ otherwise} \end{cases} \tag{4}$$

where $0 \leqq a \leqq b \leqq \pi$. In the business-cycle literature, the values of a and b are often set to the frequencies that correspond to 1.5 and 8 years, respectively. Theoretically, we need an infinite number of observations, x_t's, to compute y_t. In practice, the filtering methods approximate y_t by $\hat{y}_t$ with a finite filter. Now, we briefly review three filtering methods mentioned above.

3.1　Christiano-Fitzgerald Filter

Christiano and Fitzgerald (2003) seeked an optimal linear approximation with finite sample observations. They solved a minimization problem based on the mean square error (MSE) criterion in the frequency domain: minimization of a weighted sum of differences between the ideal bandpass-filter's weights and their approximates, using a spectral density of observations as a weight. They derived optimal filter weights, assuming a difference-stationary process of observed data with a trend or a drift removed if any.

In their empirical investigations, they examined the effects of the time-varying weights, the asymmetry, and the assumption on the stochastic process. They compared variance ratios and correlations between the components extracted by the CF filters and the theoretical components based on the data generating process of observations. To evaluate the second moments of the theoretical components, they used the Riemann sum in the frequency domain. They found that the time-varying weights and the asymmetry of the filter contribute to a better approximation, pointing out that the time-varying feature is relatively more important. Further, they claimed that the time-varying weights should not introduce severe nonstationarity in the filter approximation because the variance ratios do not vary much through the time. The correlation between the filtered-out components and the theoretical ones at different leads and lags symmetrically diminishes as the leads and lags go far away, which might indicate that the degree of asymmetry is not great. Finally, a CF filter derived under the Random-Walk data generating process, the so-called Random Walk filter, gives a good approximation to the optimal filtering that explicitly uses the estimated coefficients of an optimal moving average process determined empirically. Therefore, they claimed that we could use the Random Walk filter without inspecting the data generating process even if the random walk assumption is false. In this paper, we denote it by CF (RW) henceforth.

Details of the CF filter are given in Christiano and Fitzgerald (2003) and its properties are discussed in Iacobucci and Noullez (2005). As argued in Otsu (2015), the cyclical components extracted by CF (RW) might be distorted in magnitude and timing. The *gain* function, defined as the modulus of the frequency response function, shows large ripples over the target ranges, indicating a large distortion in estimating the cyclical components. It also shows leakage effects over higher frequencies of more than 8 periods per cycle. Further, phase shifts are indicated by values of its phase function indicate, defined as arctangent of the ratio of the real-valued coefficient of the imaginary part of the frequency response function to the real part value.

3.2 Hamming-Windowed Filter

Iacobucci and Noullez (2005) claimed that the Hamming-windowed filter be a good candidate for extracting frequency-defined components. The proposed filter has a flatter response over the passband than other filters in the literature, such as the HP filter (Hodrick and Prescott, 1997), the BK filter (Baxter and King, 1999), and the CF (RW) filter. This means that it has almost no leakage and compression, and eliminates high-frequency components better than these three filters. The filtering is implemented in the frequency domain. The procedure is described as follows. First, we subtract, if necessary, the least-square regression line to detrend the observation series to make it suitable for the Fourier transform. Second, we implement the Fourier transform of the detrended series, Third, we convolve the ideal response with a spectral window to find the windowed filter response in the frequency domain. The window is the so-called Tukey-Hamming window (Priestly, 1981, pp. 433-442).

3.3 Butterworth Filters

Pollock (2000) have proposed the tangent-based Butterworth filters in the two-sided expression, which are called rational square-wave filters. The one-sided Butterworth filters are widely used in electrical engineering, and well documented in standard text books, such as Oppenheim and Schafer (1999) and Proakis and Manolakis (2007). The two-sided version guarantees phase neutrality or no phase shift. It has finite coefficients, and its frequency response is maximally flat over the pass band: the first $(2n-1)$ derivatives of the frequency response are zero at zero frequency for the nth-order filter. The filter could stationarize an integrated process of order up to $2n$. The order of the filter can be determined so that the edge frequencies of the pass band and/or the stop band are aligned to some designated frequencies. Further, Gomez (2001) pointed out that the two-sided Butterworth filters could be interpreted as a class of statistical models called UCARIMA (the unobserved components autoregressive-integrated moving average) in Harvey (1989, p. 74). Since the Butterworth filters are not so often used in the literature, we present relevant equations to look at them a little bit more closely.

The lowpass filter is expressed as

$$BFT_L = \frac{(1+L)^n(1+L^{-1})^n}{(1+L)^n(1+L^{-1})^n + \lambda(1-L)^n(1-L^{-1})^n} \tag{5}$$

where $L^d x_t = x_{t-d}$, and $L^{-d} x_t = x_{t+d}$. Similarly, the highpass filter is expressed as

$$BFT_H = \frac{\lambda(1-L)^n(1-L^{-1})^n}{(1+L)^n(1+L^{-1})^n + \lambda(1-L)^n(1-L^{-1})^n} \tag{6}$$

Note $BET_L + BFT_H = 1$, which is the complementary condition discussed by Pollock (2000, p. 321). Here, λ is the so-called smoothing parameter.

We observe that the Butterworth highpass filter in eq.(6) can handle nonstationary components integrated of order $2n$ or less. Let ω_c the *cutoff point* at which the gain is equal to 0.5. It is shown

$$\lambda = \{tan(\omega_c/2)\}^{-2n} \tag{7}$$

To see this, we replace the L by $e^{-i\omega}$ in eq.(5) to obtain the frequency response function in polar form as

$$\psi_L(e^{-i\omega};\lambda,n) = \frac{1}{1+\lambda(i(1-e^{-i\omega})/(1+e^{-i\omega}))^{2n}} \tag{8}$$

$$= \frac{1}{1+\lambda\{tan(\omega/2)\}^{2n}} \tag{9}$$

Here, it is easy to see that eq.(7) holds when $\psi_L(e^{-i\omega}) = 0.5$. We also observe in eq.(9) that the first $(2n-1)$ derivatives of $\psi_L(e^{-i\omega})$ are zero at $\omega = 0$; thus, this filter is maximally flat. Note that the gain is the modulus of the frequency response function, and indicates to what degree the filter passes the amplitude of a component at each frequency. The Butterworth filters considered here are symmetric and their frequency response functions are non-negative. Therefore, the gain is equivalent to the frequency response. Then, we can use eq.(9) to specify ω_c so that the gain at the edge of the pass band is close to one and that of the stop band close to zero. Let the pass band $(0,\omega_p]$, and the stop band $[\omega_s,\pi]$, where ω_p is smaller than ω_s. As in Gomez (2001, p. 372), we consider the following conditions for some small positive values of δ_1 and δ_2,

$$1-\delta_1 < |\psi_L(e^{-i\omega};\lambda,n)| \leqq 1 \quad \text{for} \quad \omega \in [0,\omega_p] \tag{10}$$

$$0 \leqq |\psi_L(e^{-i\omega};\lambda,n)| < \delta_2 \quad \text{for} \quad \omega \in [\omega_s,\pi] \tag{11}$$

That is, we can control leakage and compression effects with precision specified by the values of δ_1 and δ_2. These conditions can be written as follows:

$$1 + \left(\frac{tan(\omega_p/2)}{tan(\omega_c/2)}\right)^{2n} = \frac{1}{1-\delta_1} \tag{12}$$

$$1 + \left(\frac{tan(\omega_s/2)}{tan(\omega_c/2)}\right)^{2n} = \frac{1}{\delta_2} \tag{13}$$

Then, we can solve for the cutoff frequency (ω_c) and the filter's order (n), given ω_p, ω_s, δ_1 and δ_2. The closer to zeros both δ_2 and δ_2, the smaller the leakage and the compression effects. If n turns out not an integer, the nearest integer is selected.

The Butterworth filters could be based on the sine function. Instead of eq.(5) and eq.(6), the lowpass and the highpass filters can be written as follows, respectively.

$$BFS_L = \frac{1}{1+\lambda(1-L)^n(1-L^{-1})^n} \tag{14}$$

$$BFS_H = \frac{\lambda(1-L)^n(1-L^{-1})^n}{1+\lambda(1-L)^n(1-L^{-1})^n} \tag{15}$$

where

$$\lambda = \{2sin(\omega_c/2)\}^{-2n} \tag{16}$$

These are the so-called sine-based Butterworth filters. When n is equal to two, eq.(15) is the HP cyclical filter, derived in King and Rebelo (1993, p. 224). Thus, as pointed out by Gomez (2001, p. 336), the sine-based Butterworth filter with order two ($n = 2$) can be viewed as the HP filter. As in the case of the tangent-based one, the cutoff point, ω_c, can be determined with the following conditions:

$$1+\left(\frac{sin(\omega_p/2)}{sin(\omega_c/2)}\right)^{2n}=\frac{1}{1-\delta_1} \tag{17}$$

$$1+\left(\frac{sin(\omega_s/2)}{sin(\omega_c/2)}\right)^{2n}=\frac{1}{\delta_2} \tag{18}$$

We observe that the Butterworth highpass filter in eq.(6) or eq.(15) can handle nonstationary components integrated of order $2n$ or less. Thus, the HP filter can stationarize the time series with unit root components up to the fourth order. Gomez (2001, p. 367) claimed that the BFT would give better approximations to ideal low-pass filters than the BFS. A simulation study in Otsu (2007) confirmed it. In the following analysis, we use both BFT and BFS for completeness.

In the paper, we apply the Butterworth filters to extraction of components over a certain band $[\omega_1, \omega_2]$, where ω_1 is smaller than ω_2. The bandpass filter is obtained as the difference between two highpass filters in eq.(6), or two lowpass filters in eq.(5) with different values of λ, as in Baxter and King (1999, p. 578). Suppose a lowpass filter has the pass band $[0, \omega_{p1}]$ and the stop band $[\omega_1, \pi]$. Here, ω_{p1} indicates a frequency at which the cycle is longer by some periods than at ω_1 and corresponds to ω_p in eq.(12), while ω_1 corresponds to ω_s in (13). This lowpass filter has the cutoff frequency of ω_{c1} and the order of n_1 determined in eq.(12) and (13). Let λ_1 the corresponding value of λ. Similarly, another lowpass filter has the pass band $[0, \omega_2]$ and the stop band $[\omega_{p2}, \pi]$. Here, ω_{p2} indicates a frequency at which the cycle is shorter by some periods than at ω_2. In short, we assume that $\omega_{p1} < \omega_1 < \omega_2 < \omega_{p2}$. ω_2 corresponds to ω_p in (12), and ω_{p2} corresponds to ω_s in (13). The filter has the cutoff frequency of ω_{c2} and the order of n_2. Then, the value of λ is λ_2. The bandpass filter, $BFT^{bp}(\lambda_1, n_1, \lambda_2, n_2)$, can be obtained as

$$BFT^{bp}(\lambda_1, n_1, \lambda_2, n_2) = BFT_L(\lambda_2, n_2) - BFT_L(\lambda_1, n_1) \tag{19}$$

The corresponding frequency response is expressed as

$$h(\omega; \lambda_1, n_1, \lambda_2, n_2) = \psi_L(e^{-i\omega}; \lambda_2, n_2) - \psi_L(e^{-i\omega}; \lambda_1, n_1) \tag{20}$$

We can obtain the bandpass filter for the sine-type, $BFS^{bp}(\lambda_1, n_1, \lambda_2, n_2)$, and its frequency response in a similar manner.

Alternatively, we sequentially apply the highpass filter with a lower cutoff frequency to a series, and then further apply the lowpass filter with a higher cutoff frequency to the filtered series. Although Pedersen (2001, p. 1096) reported that the sequential filtering has less distorting effects than use of the linear combination of the filters, the empirical results in the following sections do not change whether we use the difference method (the linear combination) or the sequential method. Yet another method is to convert the lowpass filter to the bandpass filter by the frequency transformation, described in a standard textbook (e.g. Proakis and Manolakis, 2007, p. 733), and explicitly obtain the bandpass filter (see Gomez, 2001, p. 371). This filter, however, has only one order parameter, implicitly assuming n_1 is equal to n_2. But, the values of n_1 and n_2 are very different in fact (see Otsu, 2015). Therefore, we would not use the transformation method later in the paper.

Finally, Harvey and Trimbur (2003, pp. 248-249) derived the *generalized Butterworth bandpass filter* in the context of unobserved-component models, taking advantage of the Wiener-Kolmogorov formula. To compute the values of the smoothing parameter and the filter's order, we need determine the locational parameter values of the band and the bandwidth. Still, a numerical calculation is involved. Here, we use the difference method, because it is easy to control leakage and compression effects at a specific frequency.

Turning to implementation, we can implement the Butterworth filtering either in the time domain or in the frequency domain. Following Pollock (2000), Otsu (2007) implemented it in the time domain, and found that when the cycle period is longer than seven, the matrix inversion is so inaccurate that it is impossible to control leakage and compression effects with a certain precision specified by eq.(12) and eq.(13), or eq.(17) and eq.(18). Further, the filters at the endpoints of data have no symmetry due to the finite truncation of filters. This implies that the time-domain implementation introduces phase shifts. Therefore, we do not choose the time-domain filtering.

Alternatively, we can implement the Butterworth filtering in the frequency domain. In the frequency-domain filtering, cyclical components are computed via the inverse discrete Fourier transform, using the Fourier-transformed series with the frequency response function as their weights. In contrast to the time-domain filtering, the frequency-domain filtering does not introduce any phase shifts, as the theoretical background of the symmetrical filters dictates. For the frequency-domain procedures to work well, it is required that a linear trend be removed and circularity be preserved in the time series, which we discuss next.

3.4 Detrending Method

To obtain better estimates of cyclical components, it is desirable to remove a linear trend in the raw data. The linear regression line, recommended by Iacobucci and Noullez (2005), is often used for trend removal. As shown by Chan, Hayya, and Ord (1977) and Nelson and Kang (1981), however, this method can produce spurious periodicity when the true trend is stochastic. Another widely-used detrending method is the first differencing, which reweighs toward the higher frequencies and can distort the original periodicity, as pointed out by Baxter and King (1999), Chan, Hayya, and Ord (1977), and Pedersen (2001).

Otsu (2011) found that the drift-adjusting method employed by Christiano and Fitzgerald (2003, p. 439) could preserve the shapes of autocorrelation functions and spectra of the original data better than the linear-regression-based detrending. Therefore, this detrending method would create less distortion. Let the raw series z_t, $t = 1, \cdots, T$. Then, we compute the drift-adjusted series, x_t as follows:

$$x_t = z_t - (t + s)\hat{\mu} \tag{21}$$

where s is any integer and

$$\hat{\mu} = \frac{z_T - z_1}{T - 1} \tag{22}$$

Note that the first and the last points are the same values:

$$x_1 = x_T = \frac{Tz_1 - z_T + s(z_1 - z_T)}{T - 1} \tag{23}$$

In Christiano and Fitzgerald (2003, p. 439), s is set to -1. Although Otsu (2011) suggested some elaboration on the choice of s, it does not affect the results of our subsequent analyses in the paper. Thus, we also set s to -1.

It should be noted that the drift-adjusting procedure in eq.(21) would make the data suitable for filtering in the frequency domain. Since the discrete Fourier transform assumes circularity of data, the discrepancy in values at both ends of the time series could seriously distort the frequency-domain filtering. The eq.(23) implies that this adjustment procedure avoids such a distortionary effect.

A final remark here is that the removed trend is put back before the BB procedure is implemented. In the business cycle literature, it is important to distinguish a classical cycle and a growth one, as pointed out by Pagan (1997). The classical cycle consists of peaks and troughs in the *levels* of aggregate economic activities, often represented by the gross national product (GDP). The classical cycle is studied by Burns and Mitchell (1946), one of the influential seminal works, which found that business cycles range from 18 months (1.5 years) to 96 months (8 years) for the United States.

On the other hand, the growth cycle exists in the *detrended* series, on which the real business cycle literature focuses. The two types of the cycles show different dates of the peaks and the troughs. When a series has a cyclical component around a deterministic upward trend, typical as in economic data, detrending would make the peaks earlier, while delaying the troughs (see Bry and Boschan, 1971, p. 11). For this reason, the dating based on the growth cycle generically tends to deviate from that on the classical cycle. Then, Canova (1994) and Canova (1999) judged the estimated dates matched the official dates as long as deviations were within two or three quarters. The results in Otsu (2013) also show that the estimated dates of peaks based on the detrended series tend to mark earlier and those of troughs later than the official dates. To mitigate this deviation, the linear trend component is incorporated into input of the BB procedure.

3.5 Boundary Treatment

In addition to the detrending method, we make use of another device to reduce variations of the estimates at ends of the series: extension with a boundary treatment. As argued by Percival and Walden (2000, p. 140), it might be possible to reduce the estimates' variations at endpoints if we make use of the so-called *reflection boundary treatment* to extend the series to be filtered. We modify the *reflection boundary treatment* so that the series is extended antisymmetrically instead of symmetrically as in the conventional reflecting rule. Let the extended series f_j ,

$$f_j = \begin{cases} x_j & \text{if } 1 \leqq j \leqq T \\ 2x_1 - x_{2-j} & \text{if } -T+3 \leqq j \leqq 0 \end{cases} \tag{24}$$

That is, the $T-2$ values, folded antisymmetrically about the initial data point, are appended to the beginning of the series. We call this extension rule the *antisymmetric reflection,* distinguished from the conventional reflection.

It is possible to append them to the end of the series. The reason to append the extension at the initial point is that most filters give accurate and stable estimates over the middle range of the series. When we put the initial point in the middle part of the extended series, the starting parts of the original series would have estimates more robust to data revisions or updates than the ending parts. Since the initial data point indicates the farthest past in the time series, it does not make sense that the estimate of the initial point is subject to a large revision when additional observations are obtained in the future. Otsu (2010) observed that it moderately reduced compression effects of the Butterworth and the Hamming-windowed filters. We note that this boundary treatment makes the estimates at endpoints identically zero when a symmetric filter is applied. We filter the extended series, f_j , and extract the last T values to obtain the targeted components.

3.6　Parameter Values

To implement the Butterworth filtering, we need specify four parameter values, n_1 , n_2 , λ_1, and λ_2 in eq.(19). We obtain these values from eqs. (7), (12), and (13) for target frequency bands, that is, values of ω_p and ω_s and for given values of δ_1 and δ_2. We set both δ_1 and δ_2 to 0.01.

The precision parameter values command orders of the Butterworth filters (see Gomez, 2001). But higher orders lead to numerical instability as noted in Pollock (2000, pp. 324-325). In fact, we could not implement the Butterworth filtering in the frequency domain when we set the transition bands to the range of 55 to 56 cycles per period with a precision values of less or equal to 1%. If we allow a little bit wider transition bands, we can avoid this instability in computation.

Otsu (2015) examined width of transition bands to have numerical stability when we extract the business cycle components from the Index of Industrial Production of Japan. It is shown that the transition bands with frequencies of 12 to 18 months per cycle and 96 to 132 months per cycle give rise to the dates of peaks and troughs consistent with the officially published reference dates of the business cycles. The corresponding orders of the tangent-based Butterworth filter are 11 and 14 for each transition band under the precision value of 0.01, which are low enough to secure numerical stability.

In the paper, we attempt to extract cyclical components with periods per cycle of 1.5 years to 8 years. In terms of a period per cycle (p), a frequency (ω) is expressed as $\frac{2\pi}{p}$. Therefore, using the notation in the previous section, the target band, $[\omega_1, \omega_2]$, is $[\frac{2\pi}{96}, \frac{2\pi}{18}]$ in months. Following

Otsu (2015), we set ω_{p1} to $\frac{2\pi}{132}$ and ω_{p2} to $\frac{2\pi}{12}$. In this case, the transition bands are $[\frac{2\pi}{132}, \frac{2\pi}{96}]$ and $[\frac{2\pi}{18}, \frac{2\pi}{12}]$, respectively. Setting ω_{p1} to ω_p in eq.(12) and ω_1 to ω_s in eq.(13), we find n_1 and ω_c. λ_1 is obtained from eq.(7). Similarly, we find n_2 and λ_1 by setting ω_2 to ω_p in eq.(12) and ω_{p2} to ω_s in eq.(13), together with eq.(7). In a similar way, we compute the parameter values of the sine-based Butterworth filter from eq.(16), eq.(17) and eq.(18).

Two remarks are in order. As is always the case, the sine-based filter commands a higher order than the tangent-based on under the same precision values of δ_1 and δ_2. In addition, as already mentioned, the well-known HP filter is viewed as the sine-based Butterworth filter with an order of two. This implies that the HP filter either does not preserve the precision or requires a very wide transition band. In the literature, it is pointed out that it might mislead researchers to false empirical results (Harvey and Jaeger, 1993), or it could generate spurious business cycle dynamics (Cogley and Nason, 1995). In the paper, we use the HP filter for completeness.

4 Empirical Study

4.1 Reference Dates and Data

The reference dates of business cycles in Japan are determined by the Economic and Social Research Institute (ESRI), affiliated with the Cabinet Office, Government of Japan. The ESRI organizes the Investigation Committee for Business Cycle Indicators to inspect historical diffusion indexes compiled from selected series and other relevant information. To construct a historical diffusion index, the peaks and troughs of each individual time series are dated by the Bry-Boschan method. Thus, the reference dates correspond to those of peaks and troughs of the classical cycles, that is, the Burns-and-Mitchell-type cycle based on the level of aggregate economic activity. Typically, the final determination of the dates is made about two to three years later.

Table 2 shows the reference dates of peaks and troughs identified by the ESRI. It also contains periods of expansion, contraction, and duration of a complete cycle (trough to trough). There are 14 peak-to-trough phases identified after World War II. The average period is about 36 months for expansion, 17 for contraction, and 53 for the complete cycle. We compare the reference dates with those obtained with various filtering methods.

We use Index of Industrial Production (IIP) of Japan in monthly basis, retrieved from Nikkei NEEDS CD-ROM (2008). We use seasonally unadjusted (NEEDS series mnemonic: IIP00P001) and seasonally adjusted (NEEDS series mnemonic: IIP00P001@) series. The base year is 2000. The sample ranges from January 1955 to January 2008, 637 observations in all. Although we should examine all series composed of the coincident index, they are available only after July 1978. Here, we focus on the IIP which provides the largest sample size among the coincident-index related series of Japan. Note the sample period does not allow analyses of the peaks and troughs before January 1955 and after January 2008 in Table 2.

4.2　Comparison with Reference Dates

To assess goodness of a dating procedure, one criterion used in the literature is how well it replicates the official reference dates of the business cycle. Canova (1994) examined the estimated growth cycle to evaluate performance of 11 different detrending methods to replicate NBER dating, assuming that the detrending removes a secular component.

Similar analyses are conducted by Canova (1999) with 12 methods including Hamilton (1989)'s procedure. They found that the Hodrick-Prescott (HP) filter proposed by Hodrick and Prescott (1997) and a frequency domain filter as an approximation to the Butterworth filter (see Canova, 1998, p. 483) would be the most reliable tools to reproduce the NBER dates. Otsu (2013) also conducted a comparative analysis among bandpass filters such as the Christiano-Fitzgerald (CF) filter (Christiano and Fitzgerald, 2003), the Hamming-windowed filter (Iacobucci and Noullez, 2005) and the Butterworth filters (e.g. Gomez, 2001; Pollock, 2000), using Japanese real GDP data. It shows that the Butterworth filters give the business-cycle dates closest to the official reference dates.

In the paper, we compute dates of peaks and troughs of IIP series for various filtering methods. The dating rule follows that of the BB procedure. We examine how well estimated dates replicate the official reference dates of the business cycles. We use official seasonal-adjustment series, seasonal adjusted by the Hamming-windowed filter and the Butterworth filters, and the series bandpassed by the three filtering methods.

Since Bry and Boschan (1971) presumed to use seasonally adjusted series, a seasonal adjustment would be required for dating procedures. The official seasonal adjustment is implemented with the X12-ARIMA method. To see effects of the adjustment methods, we also use the Hamming-windowed filter, Christiano-Fitzgerald filter and the Butterworth filters to seasonally adjust the series and use them in the BB procedure. Although conventional seasonal adjustment attempts to remove seasonal frequencies only, these filters attenuate all the frequencies higher than seasonal ones. As shown in Otsu (2009), however, the adjusted data have a power spectrum in frequency domain that is identical to that of the X12-ARIMA-adjusted data. Therefore, these filters can be used for seasonal adjustment. As for the CF filter, since the algorithm provided by Christiano and Fitzgerald (2003) allows a one-sided highpass filter, seasonally adjusted series are obtained by subtracting the cyclical components shorter than or equal to 12-month cycle from the original series. Figure 1 shows the seasonally adjusted series via these three methods. They are very similar to the official seasonal-adjustment data. The difference is in that the official data show more wriggles and no drop at the end of the sample.

Table 3 shows the results based on official seasonal-adjustment data. In the second column, the data is processed through all steps in the BB procedure. We find the BB procedure is not able to identify the peaks from 1960s to 1970 and 1977. Although there is no peak in 1981 in the reference dates, it identifies October 1981 as a peak. If we use the outlier-free Spencer curve instead of the 12-month moving average in Step II, the BB procedure picks up two dates close to the 1960s' peaks: March 1962 and December 1964 in the third column. In addition, it identifies April 1995 and January 2003 that are not in the reference dates. Further, we drop the Spencer filtering in the fourth column. Then, Step I, Step II.1, and Step III are ineffective. In Step IV, the 3-month moving average is used. It is shown that more peaks are picked up: January 2005 and December 2006. We observe similar results for troughs in Table 4. The identified dates in 1990s and 2000s are rather different from the reference dates. In sum, without the 12-month moving average and/or the Spencer filtering, the turns in 1960s are detected, and the turns in 1990s and 2000s are overidentified.

To see effects of steps in the BB procedure, some steps or their combinations are skipped to identify the dates. Table 5 and Table 6 show the selected results. In these cases, we start with the outlier-free Spencer curve instead of the 12-month moving average in Step II. Any of these cannot identify peaks in 1970 and 1977, and troughs in 1971 and 1977. When Step III through Step IV.2 are skipped, the turns in 1995 are not identified. This is because a trough in 1995 is tentatively identified in August in this case, instead of September in other two cases, being removed in Step V.6. Similarly, when only Step III is skipped, the last trough is tentatively identified in April 2003, while it is in August 2003 in case of skipping Step III and IV. Then, Step V.6 removes the trough of April 2003 in the second column. These suggest that steps involving the Spencer curve and the short-term moving average do not contribute to replicating the official reference dates.

As already mentioned, the bandpass filters can be used for seasonal adjustment. In Table 7 and Table 8, all frequencies higher than the 12-month cycle are removed by the Butterworth filters. The sine-based and the tangent-based give the same results. The full BB procedure (the second column) gives a very similar result to that of the official seasonal-adjustment series. Skipping Step III only, or together with Step IV, does not change the results (the third column). Further, when all steps from III to V.2 are omitted, the final turning points are same as the tentative ones based on the 12-month moving average. Then, the date of June 2007 is additionally identified as a peak, and September 1986 is identified as a trough, instead of January 1987. But, we do not see much improvement in replicating the reference dates. In the fifth column, the moving averages within the BB procedure are not executed. Then, it not only identifies the turns in 1960s, but also overidentifies the turns in 2000s. A similar result is obtained with the Hamming-windowed filter in Table 9 and Table 10, and with the CF filter in Table 11 and Table 12.

Turning to results with the bandpassed series, we find the BB-based dates get closer to the reference dates. As in the literature, we presume that business cycles range from 18 months to 96 months. Figure 2 shows the bandpassed series. Table 13 and Table 14 show results with the Butterworth filters. The results with the sine-based one slightly change whether the BB procedure uses the moving averages or not: the turns in 1962 are detected without the moving averages. The BB procedure with the tangent-based filter gives robust results. It does not matter whether the moving averages are used or not. In fact, we could skip all steps from III to V.2 without any consideration of outlier removal if we use the Butterworth-filtered series in Step II instead of the 12-month moving average. The turns in 1977 are not yet identified, and those in 1981 are overidentified. In experimental exercises, we also examined the case that the transition bands are $[\frac{2\pi}{144}, \frac{2\pi}{96}]$ and $[\frac{2\pi}{18}, \frac{2\pi}{12}]$, and obtained similar results. The dates of peaks are identical. Those of troughs are different by one month: March vis-à-vis April in 1958, October vis-à-vis September in 1965, and December vis-à-vis November in 1982. Table 15 and Table 16 show results for other bandpass filters. With the Hamming-windowed and the CF filters, the BB procedure identifies too many turns in 2000s and the beginning of 1980s, while the HP filter misses the first two turns and one turn in 1977. In sum, the BB procedure replicates the official reference dates better with the Butterworth filtering than with other filters.

5 Discussion

This paper examines what kind of filtering would be useful for the BB procedure to replicate the official reference dates, using Index of Industrial

Production (IIP) of Japan. We use the official seasonal-adjustment series, filtering-based equivalents, and business-cycle components extracted by bandpassing. The main findings are as follows. First, the business cycle components, extracted by bandpass filters, is useful for the BB procedure to produce dates of the turns close to the reference dates. Particularly, the Butterworth-based bandpass works well. Second, the seasonally adjusted series, whether adjusted by X12-ARIMA (official data) or by a bandpass filter, cannot identify turns in 1960s, or overidentify those in 2000s. Finally, we can drop the moving averages and outlier adjustment in the BB procedure when we use the tangent-based Butterworth filter. A possible abridged version of the Bry-Boschan procedure is described in Table 17. Here, we do not bother what threshold value is appropriate to identify outliers: it is set to 3.5 in the original procedure. Moreover, we have to determine neither what kinds of moving averages to use nor length of their leads and lags. All we need to do is to determine what duration is appropriate for a minimum cycle or a minimum phase.

A couple of caveats are in order. First, it is desirable to examine robustness of the results obtained here. Analyses with different data in time and region would be helpful. Second, we might tune dating rules, so that the BB-based dating gives a closer correspondence to the official reference dates. For example, it might be necessary to include depths of the business cycles into dating rules or taking into consideration other rules discussed in Webb (1991) and Harding and Pagan (2016, pp. 26-31). Finally, although we use the official reference dates as a criterion of evaluation, we may need to reinvestigate whether official turning points are appropriate. The turns in 1977 are hardly detected in many cases. It is retrospectively recognized that the period of March 1975 to January 1977 was an upturn period without a good economic sentiment, while that of January to October 1977 was a ‘slack’ period, but not a serious slump. Further, the political turbulence during 1970s might affeet the dating decision as well as economic data. Thus, it is natural that no turns are detected in these period. These are left for the future research.

Acknowledgements　A part of this work is financially supported by *Tokubetsu Kenkyu Josei* (Seiio University) under the title of「景気動向指数の簡便推計に関する研究」.

References

Baxter, M. and R. G. King, 1999, “Measuring business cycles: Approximate band-pass filters for economic time series,” The Review of Economics and Statistics 81(4), 575-593.

Bry, G. and C. Boschan, 1971, Cyclical Analysis of Time Series: Selected Procedures and Computer Programs, Technical Paper 20, NBER, New York.

Burns, A.F. and W. C. Mitchell, 1946, Measuring Business Cycles, NBER Studies in Business Cycles No. 2, NBER, New York.

Canova, F., 1994, “Detrending and turning points,” European Economic Review 38, 614-623.

Canova, F., 1998, “Detrending and business cycle facts,” Journal of Monetary Economics 41, 475-512.

Canova, F., 1999, “Does detrending matter for the determination of the reference cycle and the selection of turning points?” Economic Journal 109, 126-150.

Canova, F., 2007, Methods for Applied Macroeconomic Research, Princeton University Press, New Jersey.

Chan, H.C., J.C. Hayya, and J.K. Ord, 1977, “A note on trend removal methods: The case of polynomial regression versus variate differencing,” Econometrica 45, 737-744.

Christiano, L.J. and T.J. Fitzgerald, 2003, “The band pass filter,” International Economic Review 44 (2), 435-465.

Cogley, T. and J.M. Nason, 1995, “Effects of the Hodrick-Prescott filter on trend and difference stationary time series, implications for business cycle research,” Journal of Economic Dynamics and Control 19, 253-278.

Gomez, V., 2001, “The use of Butterworth filters for trend and cycle estimation in economic time series,” Journal of Business and Economic Statistics 19(3), 365-373.

Hamilton, J., 1989, “A new approach to the economic analysis of nonstationary time series and the business cycle,” Econometrica 57, 357-384.

Harding, D. and A. Pagan, 2001, “Extracting, analysing and using cyclical information,” presented at CEPR/Banca d'ltalia conference on Monitoring the European Business Cycle, July 2001.

Harding, D. and A. Pagan, 2002, “Dissecting the cycles: a methodological investigation,” Journal of Monetary Economics 49, 365-381.

Harding, D. and A. Pagan, 2006, “Synchronization of cycles,” Journal of Monetary Economics 132, 59-79.

Harding, D. and A. Pagan, 2016, The econometric analysis of recurrent events in macroeconomics and finance, Princeton University Press, Princeton and Oxford.

Harvey, A.C., 1989, Forecasting, Structural Time Series Models and the Kalman filter, Cambridge University Press.

Harvey, A.C. and A. Jaeger, 1993, “Detrending, stylized facts, and the business cycle,” Journal of Applied Econometrics 8, 231-247.

Harvey, A.C. and T. Trimbur, 2003, “General model-based filters for extracting cycles and trends in economic time series,” The Review of Economics and Statistics, 85(2), 244-255.

Hodrick, R.J. and E.C. Prescott, 1997, “Post-war U.S. business cycles: An empirical investigation,” Journal of Money, Credit, and Banking 29, 1-16. Also in Discussion Paper No. 451, Department of Economics, Carnegie-Mellon University, Pittsburgh, PA, 1980.

Iacobucci, A. and A. Noullez, 2005, “A frequency selective filter for short-length time series,” Computational Economics 25, 75-102.

Kendall, M.G. and A. Stuart, 1966, The Advanced Theory of Statistics, Vol. 3, Hafner, New York.

King, R.G. and S.T. Rebelo, 1993, “Low frequency filtering and real business cycles,” Journal of Economic Dynamics and Control 17, 207-231.

Nelson, C.R. and H. Kang, 1981, “Spurious periodicity in inappropriately detrended time series,” Econometrica 49, 741-751.

Oppenheim, A.V. and R.W. Schafer, 1999, Discrete-time signal processing 2nd edition, Prentice Hall International.

Otsu, T., 2007, "Time-invariant linear filters and real GDP: A case of Japan," Seijo University Economic Papers 174, 29-47, The Economic Institute of Seijo University (Japan).

Otsu, T., 2009, "Seasonal cycle and filtering," Seijo University Economic Papers 185, 175-221, The Economic Institute of Seijo University (Japan).

Otsu, T., 2010, "Separating trends and cycles," Seijo University Economic Papers 188, 95-158, The Economic Institute of Seijo University (Japan).

Otsu, T., 2011, "Toward harmless detrending," Seijo University Economic Papers 191, 1-27, The Economic Institute of Seijo University (Japan).

Otsu, T., 2013, "Peaks and troughs of cycles and filtering methods," Seijo University Economic Papers 200, 1-65, The Economic Institute of Seijo University (Japan).

Otsu, T., 2015, "On parameter tuning of Butterworth filters," Seijo University Economic Papers 208, 1-49, The Economic Institute of Seijo University (Japan).

Pagan, A.R., 1997, "Towards an understanding of some business cycle characteristics," Australian Economic Review 30, 1-15.

Pedersen, T.M., 2001, "The Hodrick-Prescott filter, the Slutzky effect, and the distortionary effect of filters," Journal of Economic Dynamics and Control 25, 1081-1101.

Percival, D.B. and A.T. Walden, 2000, Wavelet Methods For Time Series Analysis, Cambridge University Press, Cambridge.

Pollock, D.S.G., 2000, "Trend estimation and de-trending via rational square-wave filters," Journal of Econometrics 99, 317-334.

Priestly, M.B., 1981, Spectral Analysis and Time Series, Academic Press.

Proakis, J.G. and D.G. Manolakis, 2007, Digital Signal Processing 4th edition, Pearson Prentice Hall.

Watson, M., 1994, "Business cycle durations and postwar stabilization of the U.S. economy," American Economic Review, 84(1), March, 24–46.

Webb, R., 1991, "On predicting stages of business cycles," in: K. Lahiri and G. Moore, eds., Leading economic indicators: New approaches and forecasting record, Cambridge University Press, Cambridge, 109–127.

Whittle, P., 1983, Prediction and regulation by linear least-square methods, 2nd ed. (revised), Basil Blackwell, Oxford.

Table 1 Summary of Bry-Boschan Procedure

Step	Procedure
I	Outlier-removed series (XO): The data point of the original series (X) is replaced by that of the Spencer-filtered series (XSP) if its normalized difference in absolute value is larger than or equal to 3.5.
II	Dating with 12-month moving average: 1. Moving average: Compute 12-month moving average with 6 lags and 5 leads (X12), using XO. 2. Identification of peaks and troughs: Find the maximum (peak) or the minimum (trough) of X12 values within 6-month leads and lags. 3. Enforcement of alternation: Ensure the peaks and the troughs are alternate. If not, choose a peak with a greater value and a trough with a smaller value. If the values are same, choose an earlier peak and a later trough.
III	Dating with Spencer filtering: 1. Spencer filtering: Filtering XO with the Spencer filter to obtain a series named XOSP. 2. Identification of peaks and troughs: Ensure the peaks and the troughs in Step II within ±6 months, using XOSP. Modify if necessary. 3. Enforcement of alternation: Ensure alternation as in Step II. 4. Enforcement of minimum cycle duration: Check if the duration of a peak-to-peak or trough-to-trough takes at least 15-month period. If not, choose higher peaks and lower troughs, or if equal, an earlier date for a peak and a later one for a trough.
IV	Dating with short-term moving average: 1. Spencer filtering: Use the Spencer curve of the original series (X) as the trend-cycle component, and compute the irregular component by the difference between X and the Spencer curve. Find the minimum number of months (MCD) over which the average rate of change in the trend-cycle component exceeds the average change in the irregular component. If it is less than 3 months, the MCD is set to 3, while set to 6 if more than 6 months. 2. Short-term moving average: Compute the short-term moving average (MCDX) of the original series (X) with the span of MCD obtained above. The values at the first and the last dates with missing values in leads and lags, are to set to the same values as those at the nearest dates. 3. Identification of peaks and troughs: Ensure the peaks and the troughs as in Step III within ±6 months, using MCDX series. Modify if necessary. 4. Enforcement of alternation: Check alternation as in Step II.

Table 1　Summary of Bry-Boschan Procedure (continued)

Step	Procedure
V	Dating with the original series: 1. Identification of peaks and troughs: Ensure the peaks and the troughs as in Step IV within ±4 months or MCD, whichever longer, using the original series (X). Modify if necessary. 2. Enforcement of alternation: Ensure alternation as in Step II. 3. Elimination of turns within 6 months at endpoints: Eliminate peaks and troughs within 6 months of beginning and end of series. 4. Enforcement of the first and last peak (or trough) to be extrema: Eliminate peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end. 5. Enforcement of the minimum cycle duration: Check if the peak-to-peak and the trough-to-trough cycles are less than 15 months. If not, eliminate lower peaks (or higher troughs), or if equal, a later peak and an earlier trough. 6. Enforcement of the minimum phase duration: Eliminate phases (peak to trough or trough to peak) whose duration is less than 5 months.

Table 2　Reference Dates of Business Cycles in Japan

Dates (month, year)				Number of Periods (in months)		
Peak		Trough		Expansion	Contraction	Duration
June,	1951	October,	1951	—	4	—
January,	1954	November,	1954	27	10	37
June,	1957	June,	1958	31	12	43
December,	1961	October,	1962	42	10	52
October,	1964	October,	1965	24	12	36
July,	1970	December,	1971	57	17	74
November,	1973	March,	1975	23	16	39
January,	1977	October,	1977	22	9	31
February,	1980	February,	1983	28	36	64
June,	1985	November,	1986	28	17	45
February,	1991	October,	1993	51	32	83
May,	1997	January,	1999	43	20	63
November,	2000	January,	2002	22	14	36
February,	2008	March,	2009	73	13	86

Source: *Indexes of Business Conditions,* Economic and Social Research Institute, Cabinet Office, Government of Japan, October 7, 2012.

Table 3 Comparison with Reference Dates: Peaks, IIP (official S.A.)

Official Ref. Dates		BB proc.		No MA12		No MA12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month
1957	6	1957	5	1957	5	1957	5
1961	12						
				1962	3	1962	3
1964	10			1964	12	1964	12
1970	7						
1973	11						
		1974	1	1974	1	1974	1
1977	1					1977	1
1980	2	1980	2	1980	2	1980	2
		1981	10	1981	10	1981	10
1985	6	1985	5	1985	5	1985	5
1991	2	1991	5	1991	5	1991	5
				1995	4	1995	4
1997	5	1997	5	1997	5	1997	5
2000	11	2000	12	2000	12	2000	12
				2003	1	2003	1
						2005	1
						2006	12
2008	2						

* No outliers are adjusted.

Table 4 Comparison with Reference Dates: Troughs, IIP (official S.A.)

Official Ref. Dates		BB proc.		No MA12		No MA12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month
1958	6	1958	6	1958	6	1958	6
1962	10			1962	12	1962	12
1965	10			1965	5	1965	5
1971	12						
1975	3	1975	3	1975	3	1975	3
1977	10					1977	7
		1980	8	1980	8	1980	8
		1982	10	1982	10	1982	10
1983	2						
1986	11	1986	8	1986	8	1986	8
1993	10						
		1994	1	1994	1	1994	1
				1995	9	1995	9
		1998	8	1998	8	1998	8
1999	1						
		2001	11	2001	11	2001	11
2002	1						
						2003	8
						2005	7
						2007	5
2009	3						

* No outliers are adjusted.

Table 5　Comparison with Reference Dates: Peaks, IIP (official S.A.) w/o MA12

Official Ref. Dates		Skip BB.III		Skip BB.III-IV		Skip BB.III-V.2	
Year	Month	Year	Month	Year	Month	Year	Month
1957	6	1957	5	1957	5	1957	7
1961	12						
		1962	3	1962	3	1962	3
1964	10	1964	12	1964	12	1964	12
1970	7						
1973	11					1973	12
		1974	1	1974	1		
1977	1						
1980	2	1980	2	1980	2	1980	4
		1981	10	1981	10	1981	11
1985	6	1985	5	1985	5	1985	6
1991	2	1991	5	1991	5	1991	2
		1995	4	1995	4		
1997	5	1997	5	1997	5	1997	6
2000	11	2000	12	2000	12	2000	10
						2002	11
		2003	1	2003	1		
2008	2						

Table 6　Comparison with Reference Dates: Troughs, IIP (official S.A.) w/o MA12

Official Ref. Dates		Skip BB.III		Skip BB.III-IV		Skip BB.III-V.2	
Year	Month	Year	Month	Year	Month	Year	Month
1958	6	1958	6	1958	6	1958	5
1962	10	1962	12	1962	12	1962	10
1965	10	1965	5	1965	5	1965	5
1971	12						
1975	3	1975	3	1975	3	1975	3
1977	10						
		1980	8	1980	8	1980	10
		1982	10	1982	10	1982	11
1983	2						
1986	11	1986	8	1986	8	1986	9
1993	10						
		1994	1	1994	1	1994	1
		1995	9	1995	9		
		1998	8	1998	8	1998	10
1999	1						
		2001	11	2001	11	2001	12
2002	1						
				2003	8	2003	5
2009	3						

Table 7 Comparison with Reference Dates: Peaks, IIP (S.A. by Butterworth filters)

Official Ref. Dates		BB proc.		Skip BB.III (-IV)		Skip BB.III-V.2		No X12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month	Year	Month
1957	6	1957	8	1957	8	1957	8	1957	8
1961	12								
								1962	4
1964	10								
								1965	1
1970	7								
1973	11	1973	11	1973	11	1973	11	1973	11
								1976	12
1977	1								
1980	2	1980	4	1980	4	1980	5	1980	4
		1981	11	1981	11			1981	11
						1982	1		
1985	6	1985	7	1985	7	1985	9	1985	7
1991	2	1991	1	1991	1	1991	3	1991	1
1997	5	1997	5	1997	5	1997	5	1997	5
2000	11	2000	9	2000	9	2000	9	2000	9
								2002	9
								2004	6
								2006	8
						2007	6		
2008	2								

* The III-V.2 steps in BB procedure are skipped with no outlier adjustment.

Table 8 Comparison with Reference Dates: Troughs, IIP(S.A. by Butterworth filters)

Official Ref. Dates		BB proc.		Skip BB.III (-IV)		Skip BB.III-V.2		No X12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month	Year	Month
1958	6	1958	5	1958	5	1958	5	1958	5
1962	10							1962	10
1965	10							1965	6
1971	12								
1975	3	1975	4	1975	4	1975	6	1975	4
1977	10							1977	5
		1980	12	1980	12	1980	12	1980	12
		1982	11	1982	11	1982	10	1982	11
1983	2								
1986	11					1986	9		
		1987	1	1987	1			1987	1
1993	10	1993	12	1993	12	1993	12	1993	12
		1998	11	1998	11	1998	12	1998	11
1999	1								
		2001	12	2001	12			2001	12
2002	1					2002	1		
								2003	3
								2004	12
2009	3								

* The III-V.2 steps in BB procedure are skipped with no outlier adjustment.

Table 9　Comparison with Reference Dates: Peaks, IIP (S.A. by Hamming filter)

Official Ref. Dates		BB proc.		Skip BB.III (-IV)		Skip BB.III-V.2		No X12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month	Year	Month
1957	6	1957	7	1957	7	1957	8	1957	7
1961	12								
								1962	4
1964	10								
								1965	1
1970	7								
1973	11	1973	11	1973	11	1973	11	1973	11
								1976	12
1977	1								
1980	2	1980	4	1980	4	1980	5	1980	4
		1981	11	1981	11			1981	11
						1982	1		
1985	6	1985	7	1985	7	1985	9	1985	7
1991	2	1991	3	1991	3	1991	3	1991	3
1997	5	1997	6	1997	6	1997	5	1997	6
2000	11	2000	10	2000	10	2000	9	2000	10
								2002	10
								2004	5
		2007	7	2007	7	2007	6	2007	7
2008	2								

* The III-V.2 steps in BB procedure are skipped with no outlier adjustment.

Table 10　Comparison with Reference Dates: Troughs, IIP (S.A. by Hamming filter)

Official Ref. Dates		BB proc.		Skip BB.III (-IV)		Skip BB.III-V.2		No X12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month	Year	Month
1958	6	1958	4	1958	4	1958	5	1958	4
1962	10							1962	10
1965	10							1965	6
1971	12								
1975	3	1975	4	1975	4	1975	6	1975	4
1977	10							1977	5
		1980	12	1980	12	1980	12	1980	12
		1982	11	1982	11	1982	10	1982	11
1983	2								
1986	11					1986	9		
		1987	1	1987	1			1987	1
1993	10	1993	12	1993	12	1993	12	1993	12
		1998	12	1998	12	1998	12	1998	12
1999	1								
		2001	12	2001	12			2001	12
2002	1					2002	1		
								2003	4
								2004	10
2009	3								

* The III-V.2 steps in BB procedure are skipped with no outlier adjustment.

Table 11 Comparison with Reference Dates: Peaks, IIP (S.A. by CF filter)

Official Ref. Dates		BB proc.		Skip BB.III (-IV)		Skip BB.III-V.2		No X12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month	Year	Month
1957	6	1957	5	1957	5	1957	8	1957	5
1961	12								
								1962	4
1964	10								
								1965	4
1970	7								
1973	11	1973	9	1973	9	1973	11	1973	9
1977	1								
1980	2	1980	5	1980	5	1980	5	1980	5
		1981	9	1981	9			1981	9
						1982	1		
1985	6	1985	7	1985	7	1985	9	1985	7
1991	2	1991	6	1991	6	1991	3	1991	6
1997	5	1997	6	1997	6	1997	5	1997	6
2000	11	2000	9	2000	9	2000	9	2000	9
								2002	9
								2004	8
2008	2								

* The III-V.2 steps in BB procedure are skipped with no outlier adjustment.

Table 12 Comparison with Reference Dates: Troughs, IIP (S.A. by CF filter)

Official Ref. Dates		BB proc.		Skip BB.III (-IV)		Skip BB.III-V.2		No X12, No Spencer*	
Year	Month	Year	Month	Year	Month	Year	Month	Year	Month
1958	6	1958	8	1958	8	1958	5	1958	8
1962	10							1962	10
1965	10							1965	9
1971	12								
1975	3	1975	3	1975	3	1975	6	1975	3
1977	10								
		1980	12	1980	12	1980	12	1980	12
		1982	12	1982	12	1982	10	1982	12
1983	2								
1986	11					1986	9		
		1987	1	1987	1			1987	1
1993	10					1993	12		
		1994	1	1994	1			1994	1
		1998	12	1998	12	1998	12	1998	12
1999	1								
		2001	12	2001	12			2001	12
2002	1					2002	1		
								2003	3
								2005	2
2009	3								

* The III-V.2 steps in BB procedure are skipped with no outlier adjustment.

Table 13　Comparison with Reference Dates: Peaks, IIP (Bandpassed by Butterworth)

Official Ref. Dates		Sine-based filter				Tangent-based filter	
		BB proc.		No MA12, No Spencer*		Any combination**	
Year	Month	Year	Month	Year	Month	Year	Month
1957	6	1957	8	1957	8	1957	8
1961	12						
				1962	4	1962	4
1964	10	1964	11	1964	11	1964	11
1970	7	1970	8	1970	8	1970	8
1973	11	1973	11	1973	11	1973	11
1977	1						
1980	2	1980	3	1980	3	1980	3
		1981	11	1981	11	1981	11
1985	6	1985	3	1985	3	1985	3
1991	2	1991	3	1991	3	1991	3
1997	5	1997	6	1997	6	1997	6
2000	11	2000	10	2000	10	2000	10
		2007	7	2007	7	2007	7
2008	2						

* Same results without III-V.2 steps in BB procedure.
** Same results with or without filtering, or skipping III-V.2 steps. No outliers detected.

Table 14 Comparison with Reference Dates: Troughs, IIP (Bandpassed by Butterworth)

Official Ref. Dates		Sine-based filter				Tangent-based filter	
		BB proc.		No MA12, No Spencer*		Any combination**	
Year	Month	Year	Month	Year	Month	Year	Month
1958	6	1958	4	1958	4	1958	4
1962	10			1962	9	1962	9
1965	10	1965	9	1965	9	1965	9
1971	12	1971	7	1971	7	1971	6
1975	3	1975	5	1975	5	1975	5
1977	10						
		1981	1	1981	1	1981	1
		1982	11	1982	11	1982	11
1983	2						
1986	11						
		1987	2	1987	2	1987	2
1993	10	1993	11	1993	11	1993	11
		1998	10	1998	10	1998	10
1999	1						
		2001	11	2001	11	2001	12
2002	1						
2009	3						

* Same results without III-V.2 steps in BB procedure.
** Same results with or without filtering, or skipping III-V.2 steps. No outliers detected.

Table 15 Comparison with Reference Dates: Peaks, IIP (Bandpassed by filters)

Official Ref. Dates		Hamming filter		CF filter		HP filter	
Year	Month	Year	Month	Year	Month	Year	Month
1957	6	1957	9	1957	9		
1961	12						
		1962	3	1962	3		
1964	10	1964	12			1964	12
				1965	1		
1970	7	1970	7	1970	6	1970	8
1973	11	1973	12	1973	12	1973	11
1977	1	1977	1	1977	1		
1980	2	1980	2	1980	2	1980	4
						1981	11
		1982	1	1982	1		
1985	6	1985	3	1985	3	1985	5
1991	2	1991	5	1991	5	1991	6
1997	5	1997	6	1997	6	1997	6
2000	11	2000	10	2000	10	2000	10
		2002	12				
		2004	6	2004	6	2004	7
		2007	6	2007	7		
2008	2						

Note: same results with or without use of MA 12, Spencer curve, and III-V.2 steps.

Table 16 Comparison with Reference Dates: Troughs, IIP (Bandpassed by filters)

Official Ref. Dates		Hamming filter		CF filter		HP filter	
Year	Month	Year	Month	Year	Month	Year	Month
1958	6	1958	4	1958	3		
1962	10	1962	8	1962	8		
1965	10	1965	8	1965	7	1965	8
1971	12	1971	9	1971	9	1971	8
1975	3	1975	4	1975	4	1975	4
1977	10	1977	6	1977	6		
						1980	12
		1981	1	1981	1		
		1982	10	1982	10	1982	12
1983	2						
1986	11						
		1987	2	1987	2	1987	1
1993	10	1993	10	1993	9	1993	10
		1998	9	1998	9	1998	9
1999	1						
		2001	12	2001	11	2001	12
2002	1						
		2003	5				
		2005	3	2005	4	2005	1
2009	3						

Note: same results with or without use of MA 12, Spencer curve, and III-V.2 steps.

Table 17　Abridged Bry-Boschan Procedure

Step	Procedure
I	Business cycle extraction : Extracting business-cycle components by a two-sided tangent-based Butterworth filter. The deterministic linear trend is put back as described in Section 3.4.
II	Dating with the business-cycle components: 1. Identification of peaks and troughs: Find the maximum (peak) or the minimum (trough) of the extracted components within ±6 months (leads and lags). 2. Enforcement of alternation: Ensure the peaks and the troughs are alternate. If not, choose a peak with a greater value and a trough with a smaller value. If the values are same, choose an earlier peak and a later trough. 3. Elimination of turns within 6 months at endpoints: Eliminate peaks and troughs within 6 months of beginning and end of series. 4. Enforcement of the first and last peak (or trough) to be extrema: Eliminate peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end. 5. Enforcement of the minimum cycle duration: Check if the peak-to-peak and the trough-to-trough cycles are less than 15 months. If not, eliminate lower peaks (or higher troughs), or if equal, a later peak and an earlier trough. 6. Enforcement of the minimum phase duration: Eliminate phases (peak to trough or trough to peak) whose duration is less than 5 months.

Figure 1 Seasonally Adjusted Data: Index of Industrial Production of Japan

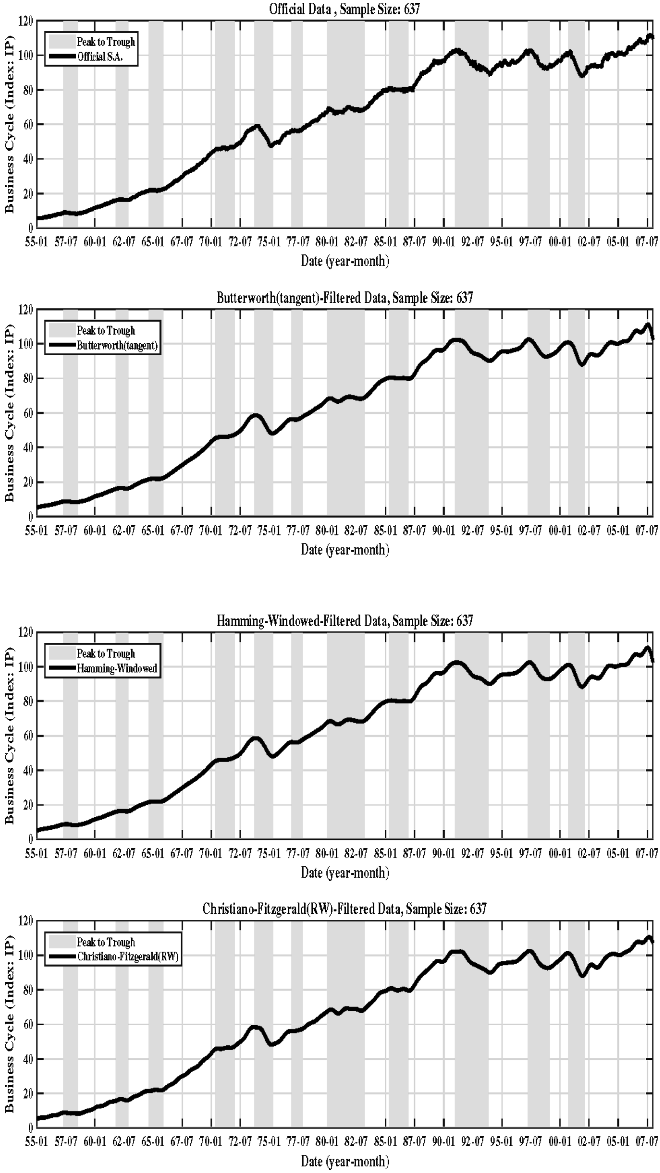

Figure 2　Bandpassed Data: Index of Industrial Production of Japan