# Robust Statistics and Business Cycle Dating

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**Abstract**

In this paper, we investigate the effects of robust statistics introduced by Economic and Social Research Institute (Japan) on the coincident composite index. Particularly, we take up interquartile ranges and median-based trimmed mean to see how these statistics affect business cycle dating results. Further, we examine what kinds of smoothing methods might make improvement in dating the business cycle in that they produce closer dates to the official reference dates. The main findings are as follows. First, the median-based outlier removing procedure and interquartile ranges play only a minor role in dating peaks and troughs. Secondly, we find that some filtering methods can simplify the Bry-Boschan algorithm to a great extent and supersede the Spencer smoothing and the short-term moving average. Thirdly, the Butterworth filter is most useful in identifying the peaks and the troughs of business cycles and nullifies necessity of introduction of the robust statistics and the ad hoc smoothing procedures originally embedded in the Bry-Boschan procedure.

*Key words*: composite index, coincident indicators; reference dates

*JEL classification*: E32

## 1 Introduction

Diffusion and composite indices are frequently used in empirical analyses of the business cycle. Each index consists of leading, coincident, and lagging indicators. The number of indicators to create composite indices is different agent by agent around the world. For example, Economic and Social Research Institute (ESRI) in Japan currently uses 9 series to calculate the coincident index, while 4 series are selected at the Conference Board in the United States (the Conference Board, 2001, p. 49).

The diffusion indices are used to determine turning points of the business cycle, that is, extract the reference cycle (Harding and Pagan, 2016, p. 52). The construction of the indices proceeds as follows. First, we determine the states of expansion and contraction. For example, we might look at the changes of each series for the diffusion indices. Then, we count the number of positive (or negative) changes at each point of time, and divide it by the total number of the series in use. If more than 50% of the series indicate positive (or negative) changes, the economic state is considered in an expansion (or contraction, respectively). Thus, the trough (or peak) can be found at a time, say, $t$, right before the share of the positive (negative) changes has just gone beyond 50% at time $t+1$.

While the diffusion indices play an important role in determination of the turning points, the composite indices would be appropriate for analyses of the amplitude of the cycles, rate of change in the cycles, and forecasting due to their construction. Since April 2008, ESRI in Japan has attached great importance to the latter, because they could be more informative for economic analyses. It also has introduced robust statistics into the composite-index compilation to reduce susceptibility to possible outliers.

Here, a question is whether the composite indices exhibit business cycle dates reasonably well in the sense that they are consistent with the official reference dates. As noted in Romer and Romer (2019), the chronology is still important in the business cycle study. Then, for the composite indices to be useful, they are expected to give the turning points of the business cycle dates well aligned with the official reference dates. Specifically, if the coincident composite index accurately follows the reference cycle, we can use it to understand the amplitude of the business cycle and the magnitude of plunges or booms. Further, it may give a criterion of validity of the business cycle models and the related econometric models.

This paper investigates the effects of robust statistics introduced by ESRI on the coincident composite index. Particularly, we take up median-based trimmed mean and interquartile ranges to see how these statistics affect business cycle dating results. Further, we examine whether smoothing methods might make improvement in dating the business cycle in that they produce closer dates to the reference dates. We start empirical analyses by examining how accurately the coincident composite index traces the reference cycle, using the business cycle data of Japan. Then, we investigate aggregation methods to compute the composite index with individual indicators in terms of use of the robust statistics, smoothing methods, and dating algorithm. Although the coincident indicators are not much used in academic research, exceptions include Stock and Watson (1991), which estimated the so-called single-index model for the U. S. with four coincident indicators and compared the estimates with the composite coincident indicator, and Harding and Pagan (2006) to investigate whether their proposed algorithm could replicate the NBER (National Bureau of Economic Research) reference cycle. Since these and other related literature typically study the closeness to the official dates, we follow suit.

The main findings are as follows. First, although ESRI uses an outlier trimming procedure and interquartile ranges for a scaling measure, these devices play only a minor role in dating peaks and troughs. Secondly, when filtering methods are used to seasonally adjust series, the Bry-Boschan algorithm can be simplified to a great extent and does not need the Spencer smoothing, the 12-month moving averages, and a trimming procedure. Thirdly, the Butterworth filter is most useful in identifying the peaks and the troughs of business cycles and nullifies necessity of introduction of the robust statistics and the ad hoc smoothing procedures originally embedded in the Bry-Boschan procedure.

The rest of the paper is organized as follows. In section 2, we summarize the aggregation method of ESRI to make the composite coincident index. Section 3 explains the algorithm of the Bry-Boschan procedure to date peaks and troughs. It is followed by a brief review of filtering methods used in the paper in section 4. In section 5, we analyze the coincident indices of Japan. The final section is allocated to discussion.

## 2 Composite Indices: ESRI Method

In this section, we explain how ESRI (Economics and Social Research Institute) computes the composite indices: the leading, the coincident and the lagging index (Economic and Social Research Institute, 2015). Its method consists of outlier removal and aggregation, and is common to the three indices. Let $x_i^j(t)$, where $j$ indicates a type of composite indices ($j=L$: leading, $C$: coincident, $Lag$: lagging), $i$ a series used in each index ($i=1, 2, \cdots, n^j$, where $n^j$ is the number of the series used for the $j$ composite index), and $t$ a point of time. Then, we first calculate a symmetric rate of change as follows:

*Step I: Symmetric Rate of Change*

To begin with, we compute the following symmetric rate of change:

$$r_i^j(t)=200\times\frac{x_i^j(t)-x_i^j(t-1)}{x_i^j(t)+x_i^j(t-1)} \tag{1}$$

*Step II: Outlier Removal*

Next, outliers are removed. Let $Q3_i^j-Q1_i^j$ an interquartile range of $r_i^j(t)$, and outlier-adjusted rate of change $\psi_i^j(t)$

$$\psi_i^j(t)=\begin{cases} -k(Q3_i^j-Q1_i^j), & \text{if } \dfrac{r_i^j(t)}{Q3_i^j-Q1_i^j}<-k \\ r_i^j(t), & \text{if } -k\leqq\dfrac{r_i^j(t)}{Q3_i^j-Q1_i^j}\leqq k \\ k(Q3_i^j-Q1_i^j), & \text{if } k<\dfrac{r_i^j(t)}{Q3_i^j-Q1_i^j} \end{cases} \tag{2}$$

Here, $k$ is a constant threshold value that is set so as to trim 5% at edges of $r_i^j(t)$ $(j=C)$, where $t$ ranges from January 1980 to the latest December. $k$ takes a value of 2.02 as of November 2011. This rule is used to remove outliers in the computational procedure up until August 2011.

Since September 2011, a refinement has been introduced in this step. The basic idea is to make outlier adjustments only to series-specific parts of the standardized rate of change, so that a shock common to all the series should be excluded from outlier removal. First, a time trend for each series is calculated as

$$m_i^j(t)=\frac{1}{60-s}\sum_{\tau=t-59}^{t-s} r_i^j(\tau) \tag{3}$$

where $s$ is the number of missing values in the summation. Then, the rate of change is standardized with the interquartile range:

$$\eta_i^j(t)=\frac{r_i^j(t)-m_i^j(t)}{Q3_i^j-Q1_i^j} \tag{4}$$

To compute the common shocks, median values are used. Let the median of $\eta_i^j(t)$ across series denoted by $\tilde{\eta}^j(t)$. Now, subtracting $\tilde{\eta}^j(t)$ from both sides of eq. (4) and rearranging it, we obtain:

$$\begin{aligned} r_i^j(t) &= \underbrace{(\eta_i^j(t)-\tilde{\eta}^j(t))(Q3_i^j-Q1_i^j)+m_i^j(t)}_{\text{specific parts}}+\underbrace{\tilde{\eta}^j(t)(Q3_i^j-Q1_i^j)}_{\text{common part}} \\ &= \hat{r}_i^j(t)+\tilde{r}_i^j(t) \end{aligned} \tag{5}$$

Then, the outlier adjustment is applied to $\hat{r}_i^j(t)$ as follows:

$$\hat{\psi}_i^j(t)=\begin{cases} -\hat{k}(\widehat{Q3}_i^j-\widehat{Q1}_i^j), & \text{if} \quad \dfrac{\hat{r}_i^j(t)}{\widehat{Q3}_i^j-\widehat{Q1}_i^j}<-\hat{k} \\ \hat{r}_i^j(t), & \text{if} \quad -\hat{k}\leqq\dfrac{\hat{r}_i^j(t)}{\widehat{Q3}_i^j-\widehat{Q1}_i^j}\leqq\hat{k} \\ \hat{k}(\widehat{Q3}_i^j-\widehat{Q1}_i^j), & \text{if} \quad \hat{k}<\dfrac{\hat{r}_i^j(t)}{\widehat{Q3}_i^j-\widehat{Q1}_i^j} \end{cases} \tag{6}$$

where $\widehat{Q3}_i^j-\widehat{Q1}_i^j$ an interquartile range of $\hat{r}_i^j(t)$. Further, $\hat{k}$ is a constant threshold value that is supposed to trim 5% at edges of $\hat{r}_i^j(t)$ $(j=C)$, where $t$ ranges from January 1985 to the latest December. $\hat{k}$ takes a value of 2.04 as of December 2015. Then, the outlier-free rate of change, $\psi_i^j(t)$ in eq. (2), is obtained as follows:

$$\psi_i^j(t)=\hat{\psi}_i^j(t)+\tilde{r}_i^j(t) \tag{7}$$

*Step III: Trend*

Two types of averages are used. The first one is called 'trend of individual series' by ESRI. It is an avereged outlier-free rate of change of each series in time domain, computed as follows:

$$\mu_i^j(t)=\frac{1}{60-s}\sum_{\tau=t-59}^{t-s}\psi_i^j(\tau) \tag{8}$$

where $s$ is the number of missing values. That is, it is an averaged value over the last 60 months, and considered as a kind of a trimmed mean due to the trimming procedure in the previous *Step II*. The second average is computed across series of the coincident index:

$$\overline{\mu}_i^C(t) = \frac{1}{n^C} \times \sum_{i=1}^{n^C} \mu_i^C(t) \tag{9}$$

where $n^C$ is the number of series used to compute the coincident index.

*Step IV: Standardized Rate of Change*

The rate of change is standardized with the interquartile range for each series:

$$z_i^j(t) = \frac{\phi_i^j(t) - \mu_i^j(t)}{Q3_i^j - Q1_i^j} \tag{10}$$

The average rate of change is computed except missing values of $z_i^j(t)$ as follows:

$$\overline{Z}^j(t) = \frac{1}{n^j - s^j(t)} \times \sum_{i=1}^{n^j} z_i^j(t) \tag{11}$$

where $s^j(t)$ is the number of series that have missing values at time $t$.

*Step V: Synthesis*

The overall average rate of change is computed as follows:

$$V^j(t) = \overline{\mu}_i^C(t) + \overline{Q3 - Q1}^j \times \overline{Z}^j(t) \tag{12}$$

where

$$\overline{Q3 - Q1}^j = \frac{1}{n^j} \times \sum_{i=1}^{n^j} (Q3_i^j - Q1_i^j) \tag{13}$$

*Step VI: Composite Index*

To compute a composite index, the following indexation is used:

$$I^j(t) = I^j(t-1) \times \frac{200 + V^j(t)}{200 - V^j(t)} \tag{14}$$

where the initial value of $I^j(t)$ is set to 1. Then, the composite index is obtained as follows:

$$CI^j(t) = \frac{I^j(t)}{I^j} \times 100 \tag{15}$$

where $I^j$ is an average in the base year.

## 3　Dating Algorithm: Bry-Boshcan Procedure

We use the Bry-Boschan (BB) procedure or its modified version to date business cycle. The full BB procedure is summarized in Table 1. Watson (1994) found some discrepancies between the original description by Bry and Boschan (1971) and the Fortran program they coded. The description here is modified to be consistent with the Fortran codes. The procedure presumes to use seasonally adjusted series. In Step I, outliers, if any, are replaced by the values of the Spencer curve. Here, the outliers are defined as values whose ratios to (or differences in absolute values from, depending on data) the 15-point Spencer curve are larger than 3.5 standard deviations, a threshold value chosen arbitrarily. This Spencer curve is computed as the 15-month symmetric moving average with particular weights (see Kendall and Stuart, 1966, p. 458).

Step II starts with the 12-month moving average (MA12, hereafter) of the outlier-free series. The MA12 is chosen on the ground that the Spencer curve contains too many minor fluctuations. Any date with the highest value among the 6 preceding and the 6 following months is tentatively regarded as the date of a peak. Similarly, any date with the lowest among the 6 preceding and the 6 following months is considered the date of a tentative trough. These peaks and troughs are checked for alternation. For contiguous peaks or troughs, the highest value is chosen for a peak, and the lowest for a trough. If the values are same, we set an earlier date for a peak, and a later date for a trough, respectively. Note that the MA12 filter is not symmetric: 6 lags and 5 leads. At the ends of the sample, it is an one-sided filter. Therefore, as we discuss later, it introduces phase shifts that might cause misinterpretation of timing of economic events.

In Step III, the Spencer curve of the outlier-free series is used to ensure peaks and troughs within ±6 months, because its turns are heuristically closer to those of the original series than those of MA12. If there are ties within ±6 data points on the Spencer curve, an earlier date is chosen for a peak, and a later date for a trough. After alternation check as in Step II, the duration of a peak to peak or a trough to trough (a full cycle) is enforced to be at least 15 months. If the duration is too short, the lower of two peaks or the higher of two troughs are eliminated. If the values are same, we set an earlier date for a peak, and a later date for a trough, respectively. Alternation check is conducted if any modification.

In Step IV, a further refinement is conducted with a short-term moving average, which is called MCD (Months for Cyclical Dominance) curve. The MCD is obtained as follows. First, we compute the Spencer curve of the original series, taking it as the trend-cycle component. The difference between the original series and the trend-cycle component gives the irregular component. Next, we take the ratio of the average change in the irregular component to that in the trend-cycle component. The change is computed either by the rate of change or by the difference of each component over various time spans. The MCD is the minimum number of months that gives the ratio less than 1. That is, the MCD is the shortest months that it takes for the change in the trend-cycle component to dominate that in the irregular component. The BB procedure confines the MCD between 3 and 6 months. Then, a short-term moving average is computed over the span of MCD, and used to ensure peaks and troughs within ±6 months as in Step III. Alternation is checked as in Step II if modified.

In the final step ('V'), a series of tests are conducted to determine final turns. First, the original series is used to ensure peaks and troughs within ±4 months or ±MCD, whichever is longer (denoted by 'V.1'). The second test ('V.2') is alternation check as in Step II. Third ('V.3'), any turn within less than 6 months from the ends is removed. In the fourth test ('V.4'), if the first or the last peak (or trough) takes a value smaller (or greater) than any value between it and the end of the original series, it is removed. In the program used by Watson (1994), the first and the last turns are only compared with the initial and the last data points, respectively, not with all the values between them. Although this could make a nontrivial difference, it does not change the results of the paper. Here, we follow Watson (1994).

The fifth test ('V.5') is to check if the duration of a full cycle is at least 15-month length, as in Step III. The final test ('V.6') is to check whether a phase (peak to trough or trough to peak) duration is at least 5 months. If it is less than 5 months, the two turning points are eliminated. If the violation is found at the last turning point, only the last point is removed. In later experiments, we implement the procedure with several steps skipped to see their effects.

## 4 Removing Seasonality: Filtering Methods

To examine effects of seasonal adjustment on determination of the reference cycle, we use three bandpass filters: Christiano-Fitzgerald filter, Hamming filter, and Butterworth filters. Since the bandpass filters are supposed to extract certain cycles of a signal, it can also extract cycles longer than seasonal cycles. Here, the difference between the bandpass filtering and the conventional seasonal adjustment procedures like X12-ARIMA is whether the cycles shorter than the seasonal cycles are removed or left. The conventional methods attempt to remove the seasonal cycles only, while the bandpass filtering removes all the cyclical components shorter than and equal to the seasonal cycles. Because our study concerns business cycles which are supposed to be longer than the seasonal cycles, we have no good reason to leave the shorter cycles in the series. Further, removal of the shorter cycles could give rise to denoising effects so that arbitrary outlier removal is less likely to play a great role in dating business cycles.

Before reviewing filtering methods, we first note several criteria to assess relative performance among those methods in terms of economic analyses. One criterion is whether a method can extract cyclical components to replicate official reference dates of the business cycles. Here, the cyclical components obtained by filtering are considered to be the growth cycle that is supposed to have a close relation to the business cycle. Canova (1994) examined performance of 11 different detrending methods to replicate NBER dating, assuming that the detrending removes a secular component. Similar analyses are conducted by Canova (1999) with 12 methods including Hamilton (1989)'s procedure. They found that the Hodrick-Prescott (HP) filter proposed by Hodrick and Prescott (1997) and a frequency domain filter as an approximation to the Butterworth filter (see Canova, 1998, p. 483) would be the most reliable tools to reproduce the NBER dates. Recently, Otsu (2013) conducted a comparative analysis among bandpass filters such as the Christiano-Fitzgerald filter (Christiano and Fitzgerald, 2003), the Hamming-windowed filter (Iacobucci and Noullez, 2005) and the Butterworth filters (e.g. Gomez, 2001; Pollock, 2000), using Japanese real GDP data. It showed that the Butterworth filters give the business cycle dates closest to the official reference dates.

Another criterion is *phase shift*. That is to say, detrending or transformation should cause no phase shifts so that it would not change time alignment of events. In general, use of one-sided filters or statistical models with lagged variables alone would cause phase shifts, which may lead to misinterpretation of economic events. Free from phase shifts are two-sided and symmetrical filters such as the Baxter-King (BK) filters (Baxter and King, 1999), the Hamming-windowed filter, and two-sided Butterworth filters. Since a large phase shift tends to lead to a large deviation of estimated business cycle dates from the official ones, this criterion is closely related to the first criterion.

The third criterion is stability of the estimated components, so that they would not change when more observations become available. Then, filtering procedures had better not be subject to the whole sample. Since most of the procedures involve estimation of coefficients, time-varying weights, or the Fourier transform, their resulting components would be susceptible to data updating. Therefore, it is a matter of degree. Otsu (2011b) examined stability of two types of frequency-domain filtering methods, the Hamming-windowed filter and the Butterworth filters, and one time-varying filtering method in time domain, the Christiano-Fitzgerald filtering. It found that the larger the sample size, the more stable the estimated components based on the frequency filtering, and that the sample size of 100 for quarterly data would be good enough to obtain stable estimates in practice. It also showed that the Butterworth filters give the most stable estimates among others. Thus, they might be useful in practice.

The fourth criterion is how a weight of each cyclical component alters by detrending or transformation, which is called *exacerbation* in Baxter and King (1999). When we use finite time-domain filters to approximate the ideal filter, certain components tend to be magnified or reduced as a result of filtering. To inspect this point, it is useful to look at the frequency response function of the time-domain filter. Then, it would show oscillations over the frequencies of the pass band and the stop band, indicating magnification and reduction of certain components. As the filter length gets longer, the oscillations become more rapid but do not diminish in amplitude. They converge to the band edges or the discontinuity points of the ideal filter, which is called *Gibbs phenomenon*. This phenomenon is attributed to approximation of infinite sum by truncation. This implies that cutting out a part of the Fourier-transformed series discontinuously as in Canova (1998, p. 483) would create the same artificial oscillatory behavior in the estimated components. In light of this criterion, the Butterworth filters and the Hamming-windowed filter have a desirable property because they have flat frequency response functions over the ranges of the pass band and the stop band.

The final criterion is the degree of *leakage* and *compression* as discussed in Baxter and King (1999). That is, detrending or filtering might admit substantial components from the range of frequencies that are supposed to suppress (*leakage*), and lose substantial components over the range to be retained (*compression*). Since these effects depend on the width of transition bands between the pass and the stop bands, it is better to have narrow transition bands. Otsu (2009, 2010) showed that the Butterworth filters are least afflicted with leakage and compression effects among others. In the related study, Otsu (2007) examined discrepancies between the ideal filter and several approximate filters, and found that the Butterworth filters give a better approximation than other bandpass filters. This also implies that the Butterworth filters could give rise to the least leakage, compression, and exacerbation effects.

Now we review properties of three methods used later: Christiano-Fitzgerald filter, Hamming filter, and Butterworth filters. To begin with, we consider the following orthogonal decomposition of the observed series $x_t$:

$$x_t = y_t + \tilde{x}_t \tag{16}$$

where $y_t$ is a signal whose frequencies belong to the interval $\{[-b, -a] \cup [a, b]\} \in [-\pi, \pi]$, while $\tilde{x}_t$ has the complementary frequencies. Suppose that we wish to extract the signal $y_t$. The Wiener-Kolmogorov theory of signal extraction, as expounded by Whittle (1983, Chapter 3 and 6), indicates $y_t$ can be written as:

$$y_t = B(L)x_t \tag{17}$$

$$B(L)=\sum_{j=-\infty}^{\infty} B_j L^j,\quad L^k x_t \equiv x_{t-k} \tag{18}$$

In polar form, we have

$$B(e^{-i\omega})=\begin{cases} 1, \text{ for } \omega \in [-b, -a] \cup [a, b] \\ 0, \text{ otherwise} \end{cases} \tag{19}$$

where $0 \leqq a \leqq b \leqq \pi$. Theoretically, we need an infinite number of observations, $x_t$'s, to compute $y_t$. In practice, the filtering methods approximate $y_t$ by $\hat{y}_t$, a filtered series with a finite filter. To estimate $y_t$ by $\hat{y}_t$, the Christiano-Fitzgerald filtering is performed in the time domain with truncation at both ends of the sample, while other filtering methods in the frequency domain are implemented under the circularity assumption. In application to seasonal adjustment, when we set $a$ to zero and $b$ to the seasonal frequencies concerned, we have power spectra identical to those of the seasonally adjusted series published officially (see Otsu, 2009, p. 212 and p. 219). In the following analyses, we set $b$ to $\frac{\pi}{6}$ . Now, we briefly review three filtering methods mentioned above.

### *4.1 Christiano-Fitzgerald Filter*

Christiano and Fitzgerald (2003) seeked an optimal linear approximation with finite sample observations. They solved a minimization problem based on the mean square error (MSE) criterion in the frequency domain: minimization of a weighted sum of differences between the ideal bandpass-filter's weights and their approximates, using a spectral density of observations as a weight. They derived optimal filter weights, assuming a difference-stationary process of observed data with a trend or a drift removed if any.

In their empirical investigations, they examined the effects of the time-varying weights, the asymmetry, and the assumption on the stochastic process.

They compared variance ratios and correlations between the components extracted by the Christiano-Fitzgerald filters and the theoretical components based on the data generating process of observations. To evaluate the second moments of the theoretical components, they used the Riemann sum in the frequency domain. They found that the time-varying weights and the asymmetry of the filter contribute to a better approximation, pointing out that the time-varying feature is relatively more important. Further, they claimed that the time-varying weights should not introduce severe nonstationarity in the filter approximation because the variance ratios do not vary much through the time. The correlation between the filtered-out components and the theoretical ones at different leads and lags symmetrically diminishes as the leads and lags go far away, which might indicate that the degree of asymmetry was not great. Finally, one of the Christiano-Fitzgerald filters derived under the Random-Walk data generating process, the so-called Random Walk filter, gives a good approximation to the optimal filtering that explicitly used the estimated coefficients of an optimal moving average process determined empirically. Therefore, they claimed that we could use the Random Walk filter without inspecting the data generating process even if the random walk assumption was false. In the paper, we simply denote it by CF henceforth.

Details of the CF filter are given in Christiano and Fitzgerald (2003) and its properties are discussed in Iacobucci and Noullez (2005). As argued in Otsu (2015), the cyclical components extracted by CF might be distorted in magnitude and timing. Its gain function, defined as the modulus of the frequency response function, shows large ripples over the target ranges, indicating a large distortion in estimating the cyclical components. The CF filter also shows leakage effects over higher frequencies of more than 8 periods per cycle. Further, phase shifts are indicated by values of its phase function, defined as arctangent of the ratio of the real-valued coefficient of the imaginary part of the frequency response function to the real part value.

In the paper, we first compute the cyclical components between 12-month and 2-month cycles, that is, the frequency range $\left[\frac{2\pi}{12}, \frac{2\pi}{2}\right]$, and subtract them from the original series to obtain $\widehat{y}_t$.

### *4.2 Hamming-Windowed Filter*

Iacobucci and Noullez (2005) claimed that the Hamming-windowed filter be a good candidate for extracting frequency-defined components. The proposed filter has a flatter responseover the passband than other filters in the literature, such as the HP filter (Hodrick and Prescott, 1997), the BK filter (Baxter and King, 1999), and the CF filter. This means that it has no exacerbation and eliminates high-frequency components better than the other three filters.

The Hamming-windowed filtering is implemented in the frequency domain. The procedure is described as follows. First, we subtract, if necessary, the least-square regression line to detrend the observation series to make it suitable for the Fourier transform. Second, we implement the Fourier transform of the de-trended series, Third, we convolve the ideal response with a spectral window to find the windowed filter response in the frequency domain. The window is the so-called Tukey-Hamming window (Priestly, 1981, pp. 433-442). In the paper, we compute the components with cycles longer than the 13-month cycle, that is, the frequency range $\left[0, \frac{2\pi}{13}\right]$, to obtain $\widehat{y}_t$, the seasonally-adjusted counterparts.

### *4.3 Butterworth Filters*

Pollock (2000) has proposed the tangent-based Butterworth filters in the two-sided expression, which are called rational square-wave filters. The one-sided

Butterworth filters are widely used in electrical engineering, and well documented in standard text books, such as Oppenheim and Schafer (1999) and Proakis and Manolakis (2007). The two-sided version guarantees phase neutrality or no phase shift. It has finite coefficients, and its frequency response is maximally flat over the pass band: the first $(2n-1)$ derivatives of the frequency response are zero at zero frequency for the nth-order filter. The filter could stationarize an integrated process of order up to $2n$. The order of the filter can be determined so that the edge frequencies of the pass band and/or the stop band are aligned to some designated frequencies. Further, Gomez (2001) pointed out that the two-sided Butter-worth filters could be interpreted as a class of statistical models called UCARIMA (the unobserved components autoregressive-integrated moving average) in Harvey (1989, p. 74). Since the two-sided Butterworth filters are not so often used in the literature, we present relevant equations to look at them a little bit more closely.

The lowpass filter is expressed as

$$BFT_L=\frac{(1+L)^n(1+L^{-1})^n}{(1+L)^n(1+L^{-1})^n+\lambda(1-L)^n(1-L^{-1})^n} \tag{20}$$

where $L^d x_t = x_{t-d}$, and $L^{-d} x_t = x_{t+d}$. Similarly, the highpass filter is expressed as

$$BFT_H=\frac{\lambda(1-L)^n(1-L^{-1})^n}{(1+L)^n(1+L^{-1})^n+(1-L)^n(1-L^{-1})^n} \tag{21}$$

Note $BFT_L+BFT_H=1$, which is the complementary condition discussed by Pollock (2000, p. 321). Here, $\lambda$ is the so-called smoothing parameter. We observe that the Butterworth highpass filter in eq. (21) can handle nonstationary components integrated of order $2n$ or less. Let $\omega_c$ the *cutoff point* at which the gain is equal to 0.5. It is shown

$$\lambda=\{tan(\omega_c/2)\}^{-2n} \tag{22}$$

To see this, we replace the $L$ by $e^{-i\omega}$ in eq. (20) to obtain the frequency response function in polar form as

$$\psi_L(e^{-i\omega}, \lambda, n) = \frac{1}{1+\lambda(i(1-e^{-i\omega})/(1+e^{-i\omega}))^{2n}} \tag{23}$$

$$= \frac{1}{1+\lambda\{tan(\omega/2)\}^{2n}} \tag{24}$$

Here, it is easy to see that eq. (22) holds when $\psi_L(e^{-i\omega})=0.5$. We also observe in eq. (24) that the first $(2n-1)$ derivatives of $\psi_L(e^{-i\omega})$ are zero at $\omega=0$; thus, this filter is maximally flat. Note that the gain is the modulus of the frequency response function, and indicates to what degree the filter passes the amplitude of a component at each frequency. The Butterworth filters considered here are symmetric and their frequency response functions are non-negative. Therefore, the gain is equivalent to the frequency response. Then, we can use eq. (24) to specify $\omega_c$ so that the gain at the edge of the pass band is close to one and that of the stop band close to zero. Let the pass band $[0, \omega_p]$, and the stop band $[\omega_s, \pi]$, where $\omega_p$ is smaller than $\omega_s$. As in Gomez (2001, p. 372), we consider the following conditions for some small positive values of $\delta_1$ and $\delta_2$,

$$1-\delta_1 < |\psi_L(e^{-i\omega}, \lambda, n)| \leqq 1 \quad \text{for } \omega \in [0, \omega_p] \tag{25}$$

$$0 \leqq |\psi_L(e^{-i\omega}; \lambda, n)| < \delta_2 \quad \text{for } \omega \in [\omega_s, \pi] \tag{26}$$

That is, we can control leakage and compression effects with precision specified by the values of $\delta_1$ and $\delta_2$. These conditions can be written as follows:

$$1+\left(\frac{tan(\omega_p/2)}{tan(\omega_c/2)}\right)^{2n} = \frac{1}{1-\delta_1} \tag{27}$$

$$1+\left(\frac{tan(\omega_s/2)}{tan(\omega_c/2)}\right)^{2n} = \frac{1}{\delta_2} \tag{28}$$

Then, we can solve for the cutoff frequency $(\omega_c)$ and the filter's order $(n)$, given

$\omega_p$, $\omega_s$, $\delta_1$ and $\delta_2$. The closer to zeros both $\delta_1$ and $\delta_2$, the smaller the leakage and the compression effects. If $n$ turns out not an integer, the nearest integer is selected.

The Butterworth filters could be based on the sine function. Instead of eq. (20) and eq. (21), the lowpass and the highpass filters can be written as follows, respectively.

$$BFS_L = \frac{1}{1+\lambda(1-L)^n(1-L^{-1})^n} \tag{29}$$

$$BFS_H = \frac{\lambda(1-L)^n(1-L^{-1})^n}{1+\lambda(1-L)^n(1-L^{-1})^n} \tag{30}$$

where

$$\lambda = \{2\,sin(\omega_c/2)\}^{-2n} \tag{31}$$

These are the so-called sine-based Butterworth filters. When $n$ is equal to two, eq. (30) is the HP cyclical filter, derived in King and Rebelo (1993, p. 224). Thus, as pointed out by Gomez (2001, p. 336), the sine-based Butterworth filter with order two $(n=2)$ can be viewed as the HP filter. As in the case of the tangent-based one, the cutoff point, $\omega_c$, can be determined with the following conditions:

$$1+\left(\frac{sin(\omega_p/2)}{sin(\omega_c/2)}\right)^{2n} = \frac{1}{1-\delta_1} \tag{32}$$

$$1+\left(\frac{sin(\omega_s/2)}{sin(\omega_c/2)}\right)^{2n} = \frac{1}{\delta_2} \tag{33}$$

We observe that the Butterworth highpass filter in eq. (21) or eq. (30) can handle nonstationary components integrated of order $2n$ or less. Thus, the HP filter can stationarize the time series with unit root components up to the fourth order. Gomez (2001, p. 367) claimed that the BFT would give better approximations to ideal low-pass filters than the BFS. A simulation study in Otsu

(2007) confirmed it. In the following analysis, we use BFT to extract passband components $[0, \omega_p]$, setting $\omega_p=\frac{2\pi}{13}$, with the stop band $[\omega_s, \pi]$ setting $\omega_s=\frac{2\pi}{12}$. To implement the Butterworth filtering, we need specify two parameter values, $n$ and $\lambda$, in eq. (20) or eq. (21). We obtain these values from eqs. (22), (27) and (28) for target frequency bands, that is, values of $\omega_p$ and $\omega_s$ with given values of $\delta_1$ and $\delta_2$. We set both $\delta_1$ and $\delta_2$ to 0.01. We only use BFS (2nd order) to obtain the HP-filtered passband components, setting $\omega_c=\frac{2\pi}{13}$ in eq. (31).

Turning to implementation, we can implement the Butter-worth filtering either in the time domain or in the frequency domain. Following Pollock (2000), Otsu (2007) implemented it in the time domain, and found that when the cycle period is longer than seven, the matrix inversion is so inaccurate that it is impossible to control leakage and compression effects with a certain precision specified by eq. (27) and eq. (28), or eq. (32) and eq. (33). Further, the filters at the endpoints of data have no symmetry due to the finite truncation of filters. This implies that the time-domain implementation introduces phase shifts. Therefore, we do not choose the time-domain filtering.

Alternatively, we can implement the Butterworth filtering in the frequency domain. In the frequency-domain filtering, cyclical components are computed via the inverse discrete Fourier transform, using the Fourier-transformed series with the frequency response function as their weights. In contrast to the time-domain filtering, the frequency-domain filtering does not introduce any phase shifts, as the theoretical background of the symmetrical filters dictates. For the frequency-domain procedures to work well, it is required that a linear trend be removed and circularity be preserved in the time series, which we discuss next.

### *4.4 Extracting Seasonal Components*

To obtain better estimates of cyclical components, it is desirable to remove a linear trend in the raw data. The linear regression line, recommended by Iacobucci and Noullez (2005), is often used for trend removal. As shown by Chan, Hayya, and Ord (1977) and Nelson and Kang (1981), however, this method can produce spurious periodicity when the true trend is stochastic. Another widely-used detrending method is the first difference, which reweighs toward the higher frequencies and can distort the original periodicity, as pointed out by Baxter and King (1999), Chan, Hayya, and Ord (1977), and Pedersen (2001).

Otsu (2011a) found that the drift-adjusting method employed by Christiano and Fitzgerald (2003, p. 439) could preserve the shapes of autocorrelation functions and spectra of the original data better than the linear-regression-based detrending. Therefore, this detrending method would create less distortion. Let the raw series $z_t$, $t=1, \cdots, T$. Then, we compute the drift-adjusted series, $x_t$, as follows:

$$x_t = z_t - (t+s)\hat{\mu} \tag{34}$$

where $s$ is any integer and

$$\hat{\mu} = \frac{z_T - z_1}{T-1} \tag{35}$$

Note that the first and the last points are the same values:

$$x_1 = x_T = \frac{Tz_1 - z_T + s(z_1 - z_T)}{T-1} \tag{36}$$

In Christiano and Fitzgerald (2003, p. 439), $s$ is set to $-1$. Although Otsu (2011a) suggested some elaboration on the choice of $s$, it does not affect the results of our subsequent analyses in the paper. Thus, we also set s to $-1$.

It should be noted that the drift-adjusting procedure in eq. (34) would make the data suitable for filtering in the frequency do-main. Since the discrete Fourier transform assumes circularity of data, the discrepancy in values at both ends of the time series could seriously distort the frequency-domain filtering. The eq. (36) implies that this adjustment procedure avoids such a distortionary effect.

A final remark here is that the BB procedure is implemented with trend-included series. In the business cycle literature, it is important to distinguish a classical cycle and a growth one, as pointed out by Pagan (1997). The classical cycle consists of peaks and troughs in the *levels* of aggregate economic activities, often represented by the gross national product (GDP). The classical cycle is studied by Burns and Mitchell (1946), one of the influential seminal works, which found that business cycles range from 18 months (1.5 years) to 96 months (8 years) for the United States.

On the other hand, the growth cycle exists in the *detrended* series, on which the real business cycle literature focuses. It shows different business cycle dates from those of the classical cycles. When a series has a cyclical component around a determinis-make the peaks earlier, while delaying the troughs (see Bry and Boschan, 1971, p. 11). For this reason, the dating based on the growth cycle generically tends to deviate from that on the classical cycle. Then, Canova (1994, 1999) judged that the estimated dates matched the official dates as long as deviations were within two or three quarters. The results in Otsu (2013) also show that the estimated dates of peaks based on the detrended series tend to mark earlier and those of troughs later than the official dates. Since we only suppress the cyclical components shorter than the seasonal cycle in the paper, we do not have such a deviation due to detrending.

In addition to the detrending method, we make use of another device to reduce variations of the estimates at ends of the series: extension with a boundary treatment. As argued by Percival and Walden (2000, p. 140), it might be possible to reduce the estimates' variations at endpoints if we make use of the so-called *reflection boundary treatment* to extend the series to be filtered. We modify the reflection boundary treatment so that the series is extended antisymmetrically instead of symmetrically as in the conventional reflecting rule. Let the extended series $f_j$,

$$f_j = \begin{cases} x_j & \text{if} \quad 1 \leqq j \leqq T \\ 2x_1 - x_{2-j} & \text{if} \quad -T+3 \leqq j \leqq 0 \end{cases} \tag{37}$$

That is, the $T-2$ values, folded antisymmetrically about the initial data point, are appended to the beginning of the series. We call this extension rule the *antisymmetric reflection*, distinguished from the conventional reflection.

It is possible to append them to the end of the series. The reason to append the extension at the initial point is that most filters give accurate and stable estimates over the middle range of the series. When we put the initial point in the middle part of the extended series, the starting parts of the original series would have estimates more robust to data revisions or updates than the ending parts. Since the initial data point indicates the farthest past in the time series, it does not make sense that the estimate of the initial point is subject to a large revision when additional observations are obtained in the future. Otsu (2010) observed that it moderately reduced compression effects of the Butterworth and the Hamming-windowed filters. We note that this boundary treatment makes the estimates at endpoints identically zero when a symmetric filter is applied. We filter the extended series, $f_j$, and extract the last $T$ values to obtain the targeted components, that is, seasonal adjustment factors that are subtracted from the original series to obtain the seasonally-adjusted series.

## 5 Empirical Analysis

### *5.1 Reference Dates and Data*

The reference dates of business cycles in Japan are determined by Economic and Social Research Institute (ESRI), affiliated with the Cabinet Office, Government of Japan. ESRI organizes the Investigation Committee for Business Cycle Indicators to inspect historical diffusion indexes calculated from selected series of coincident indexes and other relevant information. To make a historical diffusion index, the peaks and troughs of each individual time series are dated by the Bry-Boschan method. Thus, the reference dates correspond to those of peaks and troughs of the classical cycles, that is, the Burns-and-Mitchell-type cycle based on the level of aggregate economic activity. Typically, the final determination of the dates is made about two to three years later.

Table 3 shows the reference dates of peaks and troughs identified by ESRI. It also contains periods of expansion, contraction, and duration of a complete cycle (trough to trough). There are 15 peak-to-trough phases identified after World War II. The average period is about 36 months for expansion, 16 for contraction, and 52 for the complete cycle. We compare the reference dates with those of the growth cycles obtained by filtering methods.

ESRI routinely examines and revises composition of the indicators. Although the latest revision is made in February 2017, our data are based on the 9th revision in November 2004, adopted until September 2011, that selected 11 economic series for the coincident indicators. We use 11 composite coincident indicators of Japan in monthly basis, retrieved from Nikkei NEEDS CD-ROM (2008). Series names, as well as mnemonics, are listed in Table 2. The sample period ranges from January 1980 to January 2008, 337 observations for each series. We choose this data set for two reasons. First, it gives a fairly long time series in consistent composition of the indicators.

Secondly, it is revealed that officials at Ministry of Health, Labor and Welfare had incorrectly conducted fundamental statistical survey on labor-related conditions since 2004. Then, one of the 11 series, ‘Index of Non-Scheduled

Worked Hours,' may need correction. Our data may include possibly incorrect data for four years after 2004. It is desirable for the following analyses not only to include business cycles as many as possible but to avoid contaminated data as much as possible. This consideration leads us to focus on the sample up to January 2008 based on the 9th-revision composition.

We note that among the series, 'Operating Profits' is available only in quarterly base (end of periods) with seasonal adjustment (X12-ARIMA). We linearly extrapolate the quarterly data points to make monthly series. All the index-type data have the base year in 2000.

### *5.2 Composite Coincident Index and Replication*

To examine to what extent the composite coincident index (CCI) deviates from the reference cycle, we compare the official reference dates with the dates of peaks and troughs implied by the CCI. We use the Bry-Boschan algorithm procedure (see section 3) to identify dates of peaks and troughs of the CCI, because ESRI uses it to calculate the diffusion index that gives fundamental information to determine the reference dates. To begin with, we use the coincident indicators seasonally adjusted by the official agents, so that we can exclude influence of different seasonal adjustment on dating results.

In the first ('Official Ref. Dates') and the second ('Official CCI') columns of Table 4, we find that dates of peaks differ between the official reference cycle and the official CCI, except May 1997. The official CCI identifies November 1981 as a peak, while the official peak date indicates February 1980. Since the composition of coincident indicators is routinely revised, the set of indicators used in 1980 is different from that of the paper. This would be one reason for the discrepancy. Yet, there are other reasons as well.

As already mentioned, ESRI uses the historical diffusion indices (coincident indicators) to determine the reference cycle. However, only publicly available are the materials used at the committee after 2002 onward. Thus, we alternatively use the current diffusion index (Nikkei NEEDS CD-ROM, 2008) to examine the deviation between the reference cycle and the composite indices. We find that the index took 92 on average during February 1979 to February 1980, 90.9 in February, down to 81.8 in March and 86.4 in April. This might give rise to the official peak date of February 1980. During May 1980 to May 1981, the current diffusion index took less than 50 points. It reached 54.5 in June 1981, marked 100 in August, then down to 54.5 in December and sharply down to 18.2 in January 1981, less than 50 afterwards up until July 1982. The official CCI seems pick up these small bumps.

There would be two reasons for this discrepancy. First, the Bry-Boshcan algorithm uses the moving averages and the Spencer smoothing: the former become asymmetric at endpoints and the latter uses averages of the initial or the last four points as observations at endpoints. Thus, it may introduce distortion in dating computation. Secondly, it eliminates peaks within 6 months at endpoints in Step IV (see Table 1). Therefore, it never identifies February 1980 as a peak since our data start in January 1980. Then, we do not pay much attention to the deviation from the reference dates in early 1980s in the following analysis.

To check our aggregation program, we attempt to replicate the official CCI by aggregating the coincident indicators (seasonally adjusted series) published by the official agents, according to the procedure described in section 2. Note that the first quartile in eq. (10), $Q1_i^j$, is set to the 84th value of 336 rates of changes, $r_i^j(t)$ in eq. (1), in ascending order, and the third quartile, $Q3_i^j$, to the 253d value. The results are shown in Table 4 and Table 5. The deviation from the reference dates is 6 months for the peaks from 1985 to 2000 and 3 months for the troughs from 1983 to 2002.

The dates in the third column ('Aggr. Indicators') in Table 4 match well with the dates given by the official CCI. Only difference is observed in 1981. In the fourth column ('Aggr. Ind. (Median)'), we use eq. (3) - (7) instead of eq. (2) to see the effect of the median-based outlier removal. Then, the peak in 1997 becomes two months earlier, March instead of May. Although we find similar effects later in the paper, it is fair to say that the median-based outlier removal has only a limited role in dating the reference cycle.

As for the troughs, the results are shown in Table 5. The official CCI deviates from the reference dates in 1993 by two months and in 1999 by one month. The aggregation of the coincident in-dicators gives dating results same as the official CCI with or without the median-based outlier removal. A large deviation in the troughs is observed for the aggregated index, but this is mainly due to the deviation in the early 1980s: February 1983 versus October 1982. If we exclude it, the deviation reduces to 3 months. Therefore, it can be said that the computed index yields the dating results equivalent to those that the official CCI does.

We now examine whether the dates of the turning points depend on the location and the scaling measures in aggregation of coincident indicators. ESRI uses the 5-year averages defined in eq. (8) and the interquartile ranges to standardize the rate of change of each indicators in eq. (10). These quantities seem preferred because they are supposed to be insensitive to outliers. When we use the sample mean instead of the 5-year trend, we have the results in the second ('Interquartile' of 'Sample Mean') and the third ('Standard Deviation' of 'Sample Mean') columns in Table 6 and Table 7. These results are same as those the official CCI gives in Table 4 and Table 5. It is interesting to see that use of the sample mean with the standard deviation produces such identical results. We can say that the interquartile ranges do not give better results than the standard deviation. In addition, the fourth column ('Standard Deviation' of 'CM: 5-year Average') in Table 7 indicates that use of the 5-year trend introduces the deviation from the date of February 1983. Here, we have no evidence to encourage the use of the interquartile ranges and the 5-year trend.

### 5.3 *Effects of Smoothing and Bry-Boshcan Procedure*

ESRI uses seasonally adjusted series. The conventional seasonal adjustment attempts to remove seasonal frequencies only, leaving all the higher frequencies in the series. In terms of economic analyses, there is no sound reason that economic data should include the frequencies higher than seasonal ones. If an economic theory neither presumes effects of seasonality among economic variables nor designates specific forms of econometric models with seasonal effects, it is most likely not to intend to explain the fluctuation shorter than seasonality. Further, if the main analytical concern is about the business cycle, we may remove all the cyclical components shorter than seasonality beforehand.

In this section, we use filtering methods discussed in section 4, instead of the X12-ARIMA method used by the official agents, to remove all the frequencies higher than seasonality. The 11 coincident indicators are separately filtered and aggregated to make a composite coincident index. Otsu (2011b) examined the performance of these filtering methods and found that they are very useful in extracting the seasonal components and that the corresponding ‘seasonally-adjusted’ series are smoother than the seasonally-adjusted series with the X12 ARIMA.

In the following analyses, we use the 5-year average in eq. (8) as a central measure. Similar results are obtained when we alternatively use a sample mean. In Table 8 and Table 9, we use the tangent-based Butterworth filter. The results of ‘Case 1’ are obtained with the Bry-Boschan (BB) procedure, skipping the steps of I, III and IV in Table 1. We also note that these results are exactly same as those of the full BB procedure. This implies that the outlier removal and the Spencer smoothing have no effect on the results. The columns of 'Case 2' show the results when we further remove Step V. 1-V. 2 in the BB procedure. Thus, the data processing in the BB procedure is limited to the asymmetric 12-month moving average in this case.

In comparison of Table 8 with Table 4, we find that the Butterworth-based smoothing gives rise to business cycle dates closer to the official reference dates than the official CCI data, whether we use the interquartile ranges or the standard deviation as a scaling measure. Interestingly, 'Case 2' shows a better result, implying that the internal smoothing procedures other than 12-month moving average do more harm than good. In contrast, we see a slightly worse result in dating troughs, comparing Table 9 with Table 5. However, this might be due to aggregation of individual indicators that could be different from those used by ESRI. If we compare the results in Table 9 with the 'Aggr. indicators' in Table 5, the Butterworth smoothing still shows improvement by 2 months.

The Hamming-windowed filter produces similar results as the Butterworth filter. The second and the fourth column labelled as 'Case 1' in Table 10 show even closer dates, because the peak in 1997 is identified exactly same as in the official reference date. The results of the troughs in Table 11 are also equivalent to those in Table 9, but marginally worse due to the discrepancy either in 1986 or 1999. In addition, we find in both of Table 10 and Table 11 that the interquartile and the standard deviation do not make difference in dating. Finally, comparing the 'Case 1' with the 'Case 2' in Table 10, the'Case 2' gives the dates closer to the official reference dates. Therefore, we find no important roles of the Spencer filtering and the short-term moving average (MCD) embedded in the original BB procedure. Again, we cannot find clear evidence that the outlier removals and the interquartile ranges play an important role in dating.

Turning to the results of the Christiano-Fitzgerald filter in Table 12, we find similar results for the peak dating as in Table 8. The choice between the interquartile and the standard deviation does not make difference, while the Spencer and the MCD smoothing do more harm than good: they make the dates further deviate from the official ones. These findings are strengthened for the troughs, as shown in Table 13. We also find that the trough dating with the Christiano-Fitzgerald filter is more dependent on the choice of the scaling statistics, and the smoothing procedures, comparing with that of the Butterworth filter in Table 9.

Finally, the Hodrick-Prescott filter gives rise to the results that depend on whether we use the Spencer and the MCD smoothing or not, but not on the choice between the interquartile ranges and the standard deviation. As for the peaks, the smoothing procedures embedded in the original BB procedure make a large deviation from the reference dates. In contrast, these procedures give marginally better results for the troughs. These findings imply that the Hodrick-Prescott filter produces noisier series than other filters.

So far, we find that the interquartile ranges, the Spencer smothing, and the short-term moving average (MCD) do not play much role in dating the business cycles when we apply the smoothing methods in section 4 to the coincident composite indices. Now, we investigate the role of the outlier removal (a threshold value of 2.02). We skip Step II in section 2 and use $r_i^j(t)$ in eq. (1) for $\psi_i^j(\tau)$ in eq. (8). That is to say, we abandon the trimmed mean procedure. In addition, we use the standard deviations instead of the interquartile ranges in Step IV and V.

The results for the peaks are shown in Table 16. We find that the filtering methods give peak dates closer to the official ones than the official CCI in Table 4: the discrepancy is only 3 months. As for the troughs, the discrepancy is 6 months in Table 17 for all the filtering methods, which is larger than that of the official

CCI in Table 5. But, the third column ('Aggr. Indicators') of Table 5 suggests that this is mainly because that our data set is different from those used by ESRI. That is, the filtering methods at least produce comparable results without the robust statistics and the outlier removals.

In summary, when we use the filtering methods to make seasonal adjustment, it does not matter whether we use the interquartile ranges or the standard deviations. Secondly, the internal smoothing in the original BB procedure, such as the Spencer and the MCD, does more harm than good. Thirdly, we find that the business cycle dates get closer to the official reference dates without the median based outlier removal. Finally, the Butterworth filter, among others, gives better results in the sense that it replicates the official reference dates more closely.

## 6 Discussion

In this paper, we investigate the effects of robust statistics introduced by ESRI (Japan) on the coincident composite index. Particularly, we take up interquartile ranges and median-based trimmed mean to see how these statistics affect business cycle dating results. Further, we examine what kinds of smoothing methods might make improvement in dating the business cycle in that they produce closer dates to the official reference dates.

The main findings are as follows. First, the median-based outlier removing procedure (trimmed mean) and interquartile ranges play only a minor role in dating peaks and troughs. Secondly, we find that some filtering methods can simplify the Bry-Boschan (BB) algorithm to a great extent and supersede the Spencer smoothing and the short-term moving average (MCD). Thirdly, the Butterworth filter is most useful in identifying the peaks and the troughs of business cycles and nullifies necessity of introduction of the robust statistics and the ad hoc smoothing procedures originally embedded in the BB procedure.

These findings suggest that it is possible to simplify and clarify the compilation process of composite indices and indicate that we could use a simple frequency-domain filtering and a conventional normalization to construct a composite index. Since composite indices await various kinds of economic analyses for various purpose, simplicity and clarity in the compilation are desirable. In many cases, the so-called outliers in economic variables give some clues to understand important economic phenomena or effects of exogenous variables. Then, careless outlier removal would be harmful.

Concerning the Bry-Boschan (BB) algorithm, we can get rid of the Spencer and the short-term moving average smoothing, so that we may avoid arbitrariness accompanied by the former in determination of polynomial orders and phases shifts introduced by the latter. All what we need is to determine the minimum duration of phases and cycles and the enforcement rules of alternation of peaks and troughs. We do not need such repeated processes as in the original BB procedure to determine the dates of peaks and troughs.

Furthermore, although ESRI uses X12-ARIMA for seasonal adjustment, it involves arbitrariness in selecting ARIMA models, setting parameters, judgements on statistical significance of estimates. Moreover, different economic variables require different X12-ARIMA models. Then, X12-ARIMA could distort the relation among economic variables, as pointed out by Sims (1974) and Wallis (1974). In contrast, the filtering methods only require frequencies or periods per cycle to be preserved or suppressed, which might be given by economic analyses, and make it possible to filter any variable with exactly same parameter values.

In the paper, we have found and discussed a possibility of simplifying the aggregation and the dating algorithm to identify peaks and troughs of the business cycle. Similar findings appear in Otsu (2019). However, there are more to be done to reach final conclusion. Future research includes investigation of data in other periods and other countries, and other dating rules suggested in the literature (see Webb, 1991; Harding and Pagan, 2016).

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**Table 1**　Summary of Bry-Boschan Procedure

| Step | Procedure |
|---|---|
| I | Outlier-removed series (XO):<br>The data point of the original series (X) is replaced by that of the Spencer-filtered series (XSP) if its normalized difference in absolute value is larger than or equal to 3.5. |
| II | Dating with 12-month moving average:<br>1. Moving average:<br>Compute 12-month moving average with 6 lags and 5 leads (X12), using XO.<br>2. Identification of peaks and troughs:<br>Find the maximum (peak) or the minimum (trough) of X12 values within 6-month leads and lags.<br>3. Enforcement of alternation:<br>Ensure the peaks and the troughs are alternate. If not, choose a peak with a greater value and a trough with a smaller value. If the values are same, choose an earlier peak and a later trough. |
| III | Dating with Spencer filtering:<br>1. Spencer filtering:<br>Filtering XO with the Spencer filter to obtain a series named XOSP.<br>2. Identification of peaks and troughs:<br>Ensure the peaks and the troughs as in Step II within ±6 months, using XOSP. Modify if necessary.<br>3. Enforcement of alternation: Ensure alternation as in Step II.<br>4. Enforcement of minimum cycle duration:<br>Check if the duration of a peak-to-peak or trough-to-trough takes at least 15-month period. If not, choose higher peaks and lower troughs, or if equal, an earlier date for a peak and a later one for a trough. |
| IV | Dating with short-term moving average:<br>1. Spencer filtering:<br>Use the Spencer curve of the original series (X) as the trend-cycle component, and compute the irregular component by the difference between X and the Spencer curve.<br>Find the minimum number of months (MCD, Months of Cyclical Dominance) over which the average rate of change in the trend-cycle component exceeds the average change in the irregular component. If it is less than 3 months, the MCD is set to 3, while set to 6 if more than 6 months.<br>2. Short-term moving average:<br>Compute the short-term moving average (MCDX) of the original series (X) with the span of MCD obtained above. The values at the first and the last dates with missing values in leads and lags, are to set to the same values as those at the nearest dates.<br>3. Identification of peaks and troughs:<br>Ensure the peaks and the troughs as in Step III within ±6 months, using MCDX series. Modify if necessary.<br>4. Enforcement of alternation: Check alternation as in Step II. |
| V | Dating with the original series:<br>1. Identification of peaks and troughs:<br>Ensure the peaks and the troughs as in Step IV within ±4 months or ±MCD, whichever longer, using the original series (X). Modify if necessary.<br>2. Enforcement of alternation: Ensure alternation as in Step II.<br>3. Elimination of turns within 6 months at endpoints:<br>Eliminate peaks and troughs within 6 months of beginning and end of series.<br>4. Enforcement of the first and last peak (or trough) to be extrema:<br>Eliminate peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end.<br>5. Enforcement of the minimum cycle duration:<br>Check if the peak-to-peak and the trough-to-trough cycles are less than 15 months.<br>If not, eliminate lower peaks (or higher troughs), or if equal, a later peak and an earlier trough.<br>6. Enforcement of the minimum phase duration:<br>Eliminate phases (peak to trough or trough to peak) whose duration is less than 5 months. |

**Table 2** Coincident Indicators: Japan (9th Revision: Nov. 2004 - Sept. 2011)

| Series Name | Mnemonic (NEEDS)* |
|---|---|
| 1. Index of Industrial Production (Mining and Manufacturing) | IIP00P001(@) |
| 2. Index of Producer's Shipments (Producer Goods for Mining and Manufacturing) | IIP00S255(@) |
| 3. Large Industrial Power Consumption, mil. kwh. | CELL9(@) |
| 4. Index of Capacity Utilization Ratio (Manufacturing) | IIP00O01(@) |
| 5. Index of Non-Scheduled Worked Hours (Manufacturing) | HWINMF00 (HWINMF05@) |
| 6. Index of Producer's Shipment (Investment Goods Excluding Transport Equipment) | IIP00S204 (IIP00SINV@) |
| 7. Retail Sales Value (Change From Previous Year, %) | ZCSHVB20 (ZCSHVB20V) |
| 8. Wholesale Sales Value (Change From Previous Year, %) | ZCSHVB00 (ZCSHVB00V) |
| 9. Operating Profits, thou. mil. yen (All Industries) | ZBOAS@** |
| 10. Index of Sales in Small and Medium Sized Enterprises (Manufacturing) | SMSALE@ |
| 11. Effective Job Offer Rate (Excluding New School Graduates) | ESRAO(@) |

* "@" indicates seasonally-adjusted series.

** Only quarterly series are available. A linear-interpolation is used to obtain monthly series.

**Table 3** Reference Dates of Business Cycles in Japan

| Dates (month, year) | | | | Number of Periods (in months) | | |
|---|---|---|---|---|---|---|
| Peak | | Trough | | Expansion | Contraction | Duration |
| June, | 1951 | October, | 1951 | — | 4 | — |
| January, | 1954 | November, | 1954 | 27 | 10 | 37 |
| June, | 1957 | June, | 1958 | 31 | 12 | 43 |
| December, | 1961 | October, | 1962 | 42 | 10 | 52 |
| October, | 1964 | October, | 1965 | 24 | 12 | 36 |
| July, | 1970 | December, | 1971 | 57 | 17 | 74 |
| November, | 1973 | March, | 1975 | 23 | 16 | 39 |
| January, | 1977 | October, | 1977 | 22 | 9 | 31 |
| February, | 1980 | February, | 1983 | 28 | 36 | 64 |
| June, | 1985 | November, | 1986 | 28 | 17 | 45 |
| February, | 1991 | October, | 1993 | 51 | 32 | 83 |
| May, | 1997 | January, | 1999 | 43 | 20 | 63 |
| November, | 2000 | January, | 2002 | 22 | 14 | 36 |
| February, | 2008 | March, | 2009 | 73 | 13 | 86 |
| March, | 2012 | November, | 2012 | 36 | 8 | 44 |

Source: *Indexes of Business Conditions*, Economic and Social Research Institute, Cabinet Office, Government of Japan, July 24, 2015.

**Table 4**　Comparison with Reference Dates: Peaks, Official S.A. Series

| Official Ref. Dates | | Official CCI | | Aggr. Indicators* | | Aggr. Ind. (Median)** | |
|---|---|---|---|---|---|---|---|
| Year | Month | Year | Month | Year | Month | Year | Month |
| 1980 | 2 | | | | | | |
| | | 1981 | 11 | 1981 | 12 | 1981 | 12 |
| 1985 | 6 | 1985 | 5 | 1985 | 5 | 1985 | 5 |
| | | 1990 | 10 | 1990 | 10 | 1990 | 10 |
| 1991 | 2 | | | | | | |
| 1997 | 5 | 1997 | 5 | 1997 | 5 | 1997 | 3 |
| 2000 | 11 | 2000 | 12 | 2000 | 12 | 2000 | 12 |
| 2008 | 2 | | | | | | |
| Deviation*** | | 6 months | | 6 months | | 8 months | |

Note: *　Aggregating 11 coincident indicators, removing outliers(threshold value: 2.02).
**　Outlier removal (threshold value: 2.02) based on median values, eq. (6).
*** Deviation from the reference dates, sum of absolute values from 1985 to 2000.

**Table 5**　Comparison with Reference Dates: Troughs, Official S.A. Series

| Official Ref. Dates | | Official CCI | | Aggr. Indicators* | | Aggr. Ind. (Median)** | |
|---|---|---|---|---|---|---|---|
| Year | Month | Year | Month | Year | Month | Year | Month |
| 1977 | 10 | | | | | | |
| | | 1981 | 5 | 1981 | 5 | 1981 | 5 |
| | | | | 1982 | 10 | 1982 | 10 |
| 1983 | 2 | 1983 | 2 | | | | |
| 1986 | 11 | 1986 | 11 | 1986 | 11 | 1986 | 11 |
| 1993 | 10 | 1993 | 12 | 1993 | 12 | 1993 | 12 |
| | | 1998 | 12 | 1998 | 12 | 1998 | 12 |
| 1999 | 1 | | | | | | |
| 2002 | 1 | 2002 | 1 | 2002 | 1 | 2002 | 1 |
| 2009 | 3 | | | | | | |
| Deviation*** | | 3 months | | 7 months | | 7 months | |

Note: *　Aggregating 11 coincident indicators, removing outliers (threshold value: 2.02).
**　Outlier removal (threshold value: 2.02) based on median values, eq.(6).
*** Deviation from the reference dates, sum of absolute values from 1983 to 2002.

**Table 6** Comparison with Reference Dates: Peaks, Alternative Measures in eq. (10)

| Official Ref. Dates | | Central Measure(CM): Sample Mean | | | | CM: 5-year Average, eq. (8) | |
|---|---|---|---|---|---|---|---|
| | | Interquartile | | Standard Deviation | | Standard Deviation | |
| Year | Month | Year | Month | Year | Month | Year | Month |
| 1980 | 2 | | | | | | |
| | | 1981 | 12 | 1981 | 11 | 1981 | 11 |
| 1985 | 6 | 1985 | 5 | 1985 | 5 | 1985 | 5 |
| | | 1990 | 10 | 1990 | 10 | 1990 | 10 |
| 1991 | 2 | | | | | | |
| 1997 | 5 | 1997 | 3 | 1997 | 3 | 1997 | 3 |
| 2000 | 11 | 2000 | 12 | 2000 | 12 | 2000 | 12 |
| 2008 | 2 | | | | | | |
| Deviation* | | 8 months | | 8 months | | 8 months | |

Note: 1. Outlier removal (threshold value: 2.02) based on median values, eq. (6).
2. Same results without Step I, III & IV and Spencer smoothing in BB proc. (Table 1)
* Deviation from the reference dates, sum of absolute values from 1985 to 2000.

**Table 7** Comparison with Reference Dates: Troughs, Alternative Measures in eq. (10)

| Official Ref. Dates | | Central Measure(CM): Sample Mean | | | | CM: 5-year Trend, eq. (8) | |
|---|---|---|---|---|---|---|---|
| | | Interquartile | | Standard Deviation | | Standard Deviation | |
| Year | Month | Year | Month | Year | Month | Year | Month |
| 1977 | 10 | | | | | | |
| | | 1981 | 5 | 1981 | 5 | 1981 | 5 |
| | | 1982 | 10 | | | 1982 | 10 |
| 1983 | 2 | | | 1983 | 2 | | |
| 1986 | 11 | 1986 | 11 | 1986 | 11 | 1986 | 11 |
| 1993 | 10 | 1993 | 12 | 1993 | 12 | 1993 | 12 |
| | | 1998 | 12 | 1998 | 12 | 1998 | 12 |
| 1999 | 1 | | | | | | |
| 2002 | 1 | 2002 | 1 | 2002 | 1 | 2002 | 1 |
| 2009 | 3 | | | | | | |
| Deviation* | | 7 months | | 3 months | | 7 months | |

Note: 1. Outlier removal (threshold value: 2.02) based on median values, eq. (6).
2. Same results without Step I, III & IV and Spencer smoothing in BB proc. (Table 1)
* Deviation from the reference dates, sum of absolute values from 1983 to 2002.

**Table 8**　Seasonal Adjustment by Butterworth (tangent based) Filter: Peaks

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1980 | 2 | | | | | | | | |
| | | 1981 | 9 | 1981 | 12 | 1981 | 9 | 1981 | 12 |
| 1985 | 6 | 1985 | 7 | 1985 | 6 | 1985 | 7 | 1985 | 6 |
| | | 1990 | 11 | 1990 | 12 | 1990 | 11 | 1990 | 12 |
| 1991 | 2 | | | | | | | | |
| 1997 | 5 | 1997 | 4 | 1997 | 5 | 1997 | 4 | 1997 | 5 |
| 2000 | 11 | 2000 | 11 | 2000 | 10 | 2000 | 11 | 2000 | 10 |
| | | | | 2007 | 6 | | | 2007 | 6 |
| 2008 | 2 | | | | | | | | |
| Deviation* | | 5 months | | 3 months | | 5 months | | 3 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1　Skip I, III and IV in BB procedure (Table 1).
Case 2　Skip I and III through V.2 in BB procedure (Table 1).
*　Deviation from the reference dates, sum of absolute values from 1985 to 2000.

**Table 9**　Seasonal Adjustment by Butterworth (tangent based) Filter: Troughs

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1977 | 10 | | | | | | | | |
| | | 1981 | 1 | 1981 | 4 | 1981 | 1 | 1981 | 3 |
| | | | | 1982 | 12 | | | 1982 | 12 |
| 1983 | 2 | 1983 | 1 | | | 1983 | 1 | | |
| 1986 | 11 | 1986 | 11 | 1986 | 10 | 1986 | 11 | 1986 | 10 |
| 1993 | 10 | 1993 | 12 | 1993 | 12 | 1993 | 12 | 1993 | 12 |
| 1999 | 1 | 1999 | 2 | 1999 | 1 | 1999 | 2 | 1999 | 1 |
| | | 2001 | 12 | | | 2001 | 12 | | |
| 2002 | 1 | | | 2002 | 1 | | | 2002 | 1 |
| 2009 | 3 | | | | | | | | |
| Deviation* | | 5 months | | 5 months | | 5 months | | 5 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1　Skip I, III and IV in BB procedure (Table 1).
Case 2　Skip I and III through V.2 in BB procedure (Table 1).
*　Deviation from the reference dates, sum of absolute values from 1983 to 2002.

**Table 10** Seasonal Adjustment by Hamming-Windowed Filter: Peaks

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1980 | 2 | | | | | | | | |
| | | 1981 | 10 | 1981 | 12 | 1981 | 10 | 1981 | 12 |
| 1985 | 6 | 1985 | 7 | 1985 | 6 | 1985 | 7 | 1985 | 6 |
| | | 1990 | 11 | 1990 | 12 | 1990 | 11 | 1990 | 12 |
| 1991 | 2 | | | | | | | | |
| 1997 | 5 | 1997 | 5 | 1997 | 5 | 1997 | 5 | 1997 | 5 |
| 2000 | 11 | 2000 | 11 | 2000 | 10 | 2000 | 11 | 2000 | 10 |
| | | | | 2007 | 6 | | | 2007 | 6 |
| 2008 | 2 | | | | | | | | |
| Deviation* | | 4 months | | 3 months | | 4 months | | 3 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1985 to 2000.

**Table 11** Seasonal Adjustment by Hamming-Windowed Filter: Troughs

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1977 | 10 | | | | | | | | |
| | | 1981 | 2 | 1981 | 4 | 1981 | 2 | 1981 | 4 |
| | | | | 1982 | 12 | | | 1982 | 12 |
| 1983 | 2 | 1983 | 2 | | | 1983 | 2 | | |
| 1986 | 11 | 1986 | 10 | 1986 | 10 | 1986 | 10 | 1986 | 10 |
| 1993 | 10 | 1993 | 12 | 1993 | 12 | 1993 | 12 | 1993 | 12 |
| 1999 | 1 | 1999 | 3 | 1999 | 2 | 1999 | 3 | 1999 | 2 |
| | | 2001 | 12 | | | 2001 | 12 | | |
| 2002 | 1 | | | 2002 | 1 | | | 2002 | 1 |
| 2009 | 3 | | | | | | | | |
| Deviation* | | 6 months | | 6 months | | 6 months | | 6 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1983 to 2002.

**Table 12**　Seasonal Adjustment by Christiano-Fitzgerald Filter: Peaks

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1980 | 2 | | | | | | | | |
| | | 1981 | 9 | 1981 | 12 | 1981 | 9 | 1981 | 12 |
| 1985 | 6 | 1985 | 7 | 1985 | 6 | 1985 | 6 | 1985 | 6 |
| | | 1990 | 10 | 1990 | 12 | 1990 | 10 | 1990 | 12 |
| 1991 | 2 | | | | | | | | |
| 1997 | 5 | 1997 | 5 | 1997 | 5 | 1997 | 5 | 1997 | 5 |
| 2000 | 11 | 2000 | 11 | 2000 | 10 | 2000 | 10 | 2000 | 10 |
| 2008 | 2 | | | | | | | | |
| Deviation* | | 5 months | | 3 months | | 5 months | | 3 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1985 to 2000.

**Table 13**　Seasonal Adjustment by Christiano-Fitzgerald Filter: Troughs

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1977 | 10 | | | | | | | | |
| | | 1981 | 1 | | | 1981 | 1 | | |
| | | 1982 | 12 | 1982 | 12 | 1982 | 12 | 1982 | 12 |
| 1983 | 2 | | | | | | | | |
| 1986 | 11 | 1986 | 11 | 1986 | 10 | 1986 | 11 | 1986 | 10 |
| 1993 | 10 | | | 1993 | 12 | | | 1993 | 12 |
| | | 1994 | 1 | | | 1994 | 1 | | |
| 1999 | 1 | 1999 | 3 | 1999 | 2 | 1999 | 2 | 1999 | 1 |
| | | 2001 | 12 | | | 2001 | 12 | | |
| 2002 | 1 | | | 2002 | 1 | | | 2002 | 1 |
| 2009 | 3 | | | | | | | | |
| Deviation* | | 8 months | | 6 months | | 7 months | | 5 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1983 to 2002.

**Table 14** Seasonal Adjustment by Hodrick-Prescott Filter: Peaks

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1980 | 2 | | | | | | | | |
| | | 1981 | 10 | 1981 | 12 | 1981 | 10 | 1981 | 12 |
| 1985 | 6 | 1985 | 7 | 1985 | 6 | 1985 | 7 | 1985 | 6 |
| | | 1990 | 10 | 1990 | 12 | 1990 | 10 | 1990 | 12 |
| 1991 | 2 | | | | | | | | |
| 1997 | 5 | 1997 | 4 | 1997 | 5 | 1997 | 4 | 1997 | 5 |
| 2000 | 11 | 2000 | 10 | 2000 | 10 | 2000 | 10 | 2000 | 10 |
| | | | | 2007 | 6 | | | 2007 | 6 |
| 2008 | 2 | | | | | | | | |
| Deviation* | | 7 months | | 3 months | | 7 months | | 3 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1985 to 2000.

**Table 15** Seasonal Adjustment by Hodrick-Prescott Filter: Troughs

| Official Ref. Dates | | Scaling Meas.: Interquartile | | | | Scaling Meas.: Std. Dev. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Case 1 | | Case 2 | | Case 1 | | Case 2 | |
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1977 | 10 | | | | | | | | |
| | | | | | | 1981 | 2 | | |
| | | | | 1982 | 12 | | | 1982 | 12 |
| 1983 | 2 | 1983 | 1 | | | 1983 | 1 | | |
| 1986 | 11 | 1986 | 11 | 1986 | 10 | 1986 | 11 | 1986 | 10 |
| 1993 | 10 | 1993 | 12 | 1993 | 12 | 1993 | 12 | 1993 | 12 |
| 1999 | 1 | 1999 | 1 | 1999 | 1 | 1999 | 1 | 1999 | 1 |
| | | 2001 | 12 | | | 2001 | 12 | | |
| 2002 | 1 | | | 2002 | 1 | | | 2002 | 1 |
| 2009 | 3 | | | | | | | | |
| Deviation* | | 4 months | | 5 months | | 4 months | | 5 months | |

Note: Central measure is the past 5-year average in eq. (8).
Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1983 to 2002.

**Table 16** Dating Peaks: No Outlier Removal and No Robust Statistics

| Official Ref. Dates | | Butterworth | | Hamming | | CF Filter | | HP Filter | |
|---|---|---|---|---|---|---|---|---|---|
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1980 | 2 | | | | | | | | |
| | | 1981 | 12 | 1981 | 12 | 1981 | 12 | | |
| | | | | | | | | 1982 | 1 |
| 1985 | 6 | 1985 | 6 | 1985 | 6 | 1985 | 6 | 1985 | 6 |
| | | 1990 | 12 | 1990 | 12 | 1990 | 12 | 1990 | 12 |
| 1991 | 2 | | | | | | | | |
| 1997 | 5 | 1997 | 5 | 1997 | 5 | 1997 | 5 | 1997 | 5 |
| 2000 | 11 | 2000 | 10 | 2000 | 10 | 2000 | 10 | 2000 | 10 |
| Deviation* | | 3 months | | 3 months | | 3 months | | 3 months | |

Note: 1. The past 5-year average for central measure, standard deviation for scaling measure.
2. Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1985 to 2000.

**Table 17** Dating Troughs: No Outlier Removal and No Robust Statistics

| Official Ref. Dates | | Butterworth | | Hamming | | CF Filter | | HP Filter | |
|---|---|---|---|---|---|---|---|---|---|
| Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. | Year | Mon. |
| 1977 | 10 | | | | | | | | |
| | | | | 1981 | 3 | | | | |
| | | 1982 | 12 | 1982 | 12 | 1982 | 12 | 1982 | 11 |
| 1983 | 2 | | | | | | | | |
| 1986 | 11 | 1986 | 10 | 1986 | 10 | 1986 | 10 | 1986 | 10 |
| 1993 | 10 | 1993 | 12 | 1993 | 12 | 1993 | 12 | 1993 | 12 |
| 1999 | 1 | 1999 | 2 | 1999 | 2 | 1999 | 2 | 1999 | 2 |
| 2002 | 1 | 2002 | 1 | 2002 | 1 | 2002 | 1 | 2002 | 1 |
| Deviation* | | 6 months | | 6 months | | 6 months | | 6 months | |

Note: 1. The past 5-year average for central measure, standard deviation for scaling measure.
2. Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1983 to 2002.